

Koja životinja nam je doneła SARS?

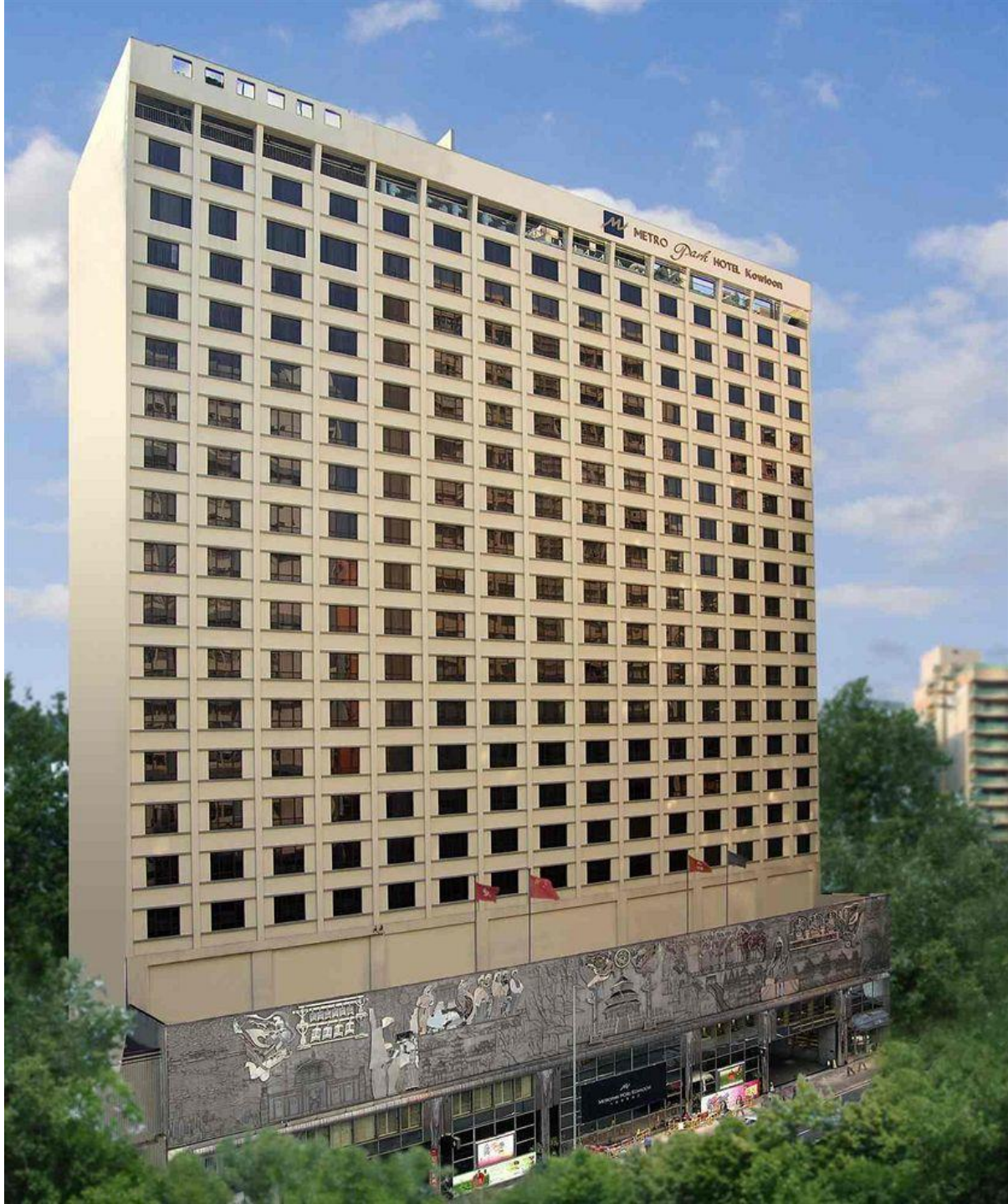
Rekonstrukcija evolutivnih stabala

*Bioinformatics Algorithms:
an Active Learning Approach*

~Poglavlje 7~

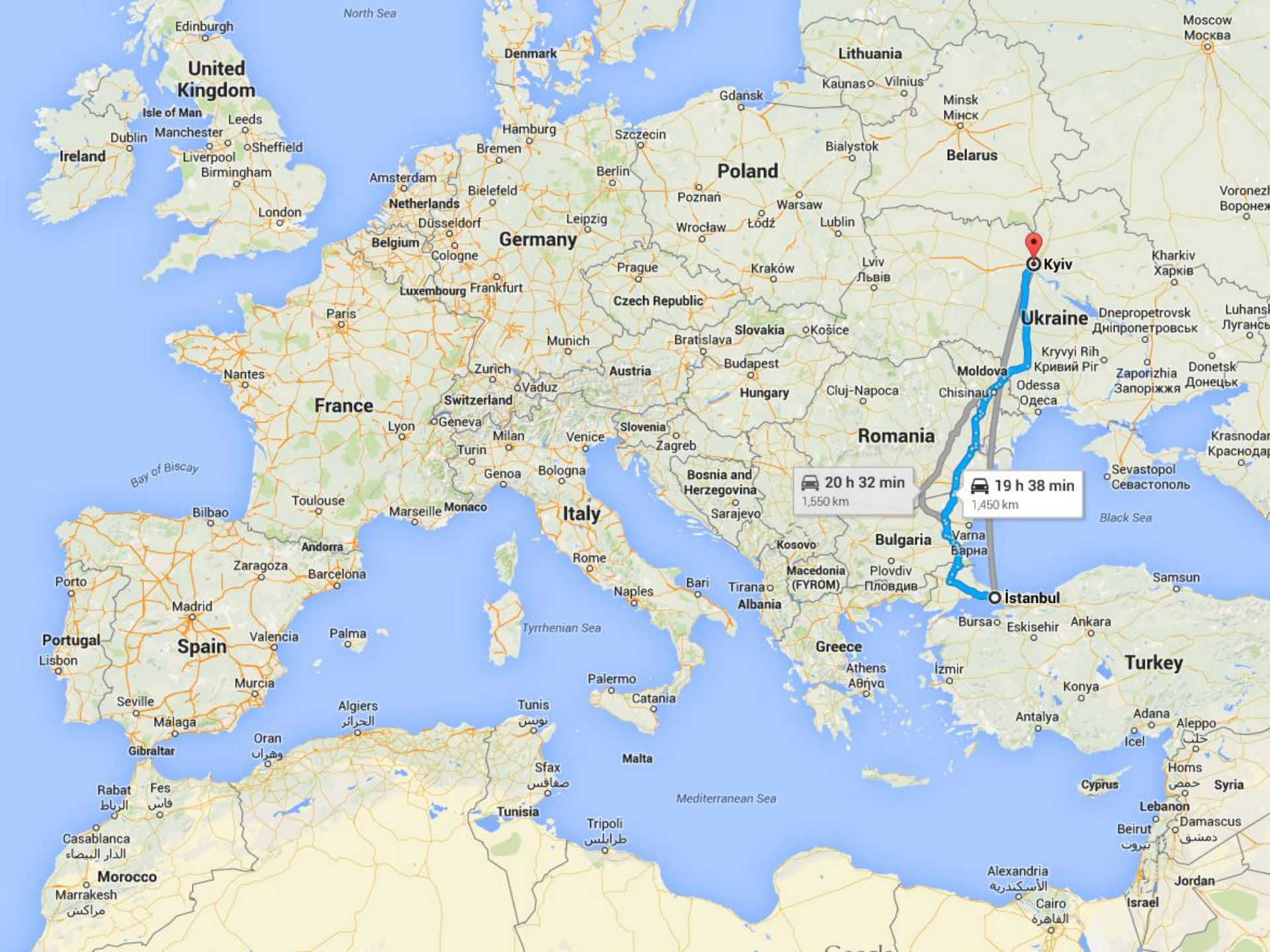
Pregled

- **Izbijanje epidemije**
- Transformacija matrice rastojanja u evolutivno stablo
- Prema algoritmu za rekonstrukciju filogenetskog stabla na osnovu rastojanja
- *AdditivePhylogeny* algoritam
- Metod najmanjih kvadrata
- Ultrametrična evolutivna stabla
- Neighbour-Joining algoritam
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- Problem male parsimonije
- Problem velike parsimonije



Modern boundaries are shown for reference.





Kyiv

20 h 32 min
1,550 km

19 h 38 min
1,450 km

Alexandria
الأسكندرية
Cairo
القاهرة



Širenje
?????



Ireland
1 case

China

Canada
29 cases
United States
1 case

Guangdong
Province

Metropole
Hotel

Hong Kong
Special
Administrative
Region
195 cases

Vietnam
58 cases

Singapore
71 cases

Arabian
Sea

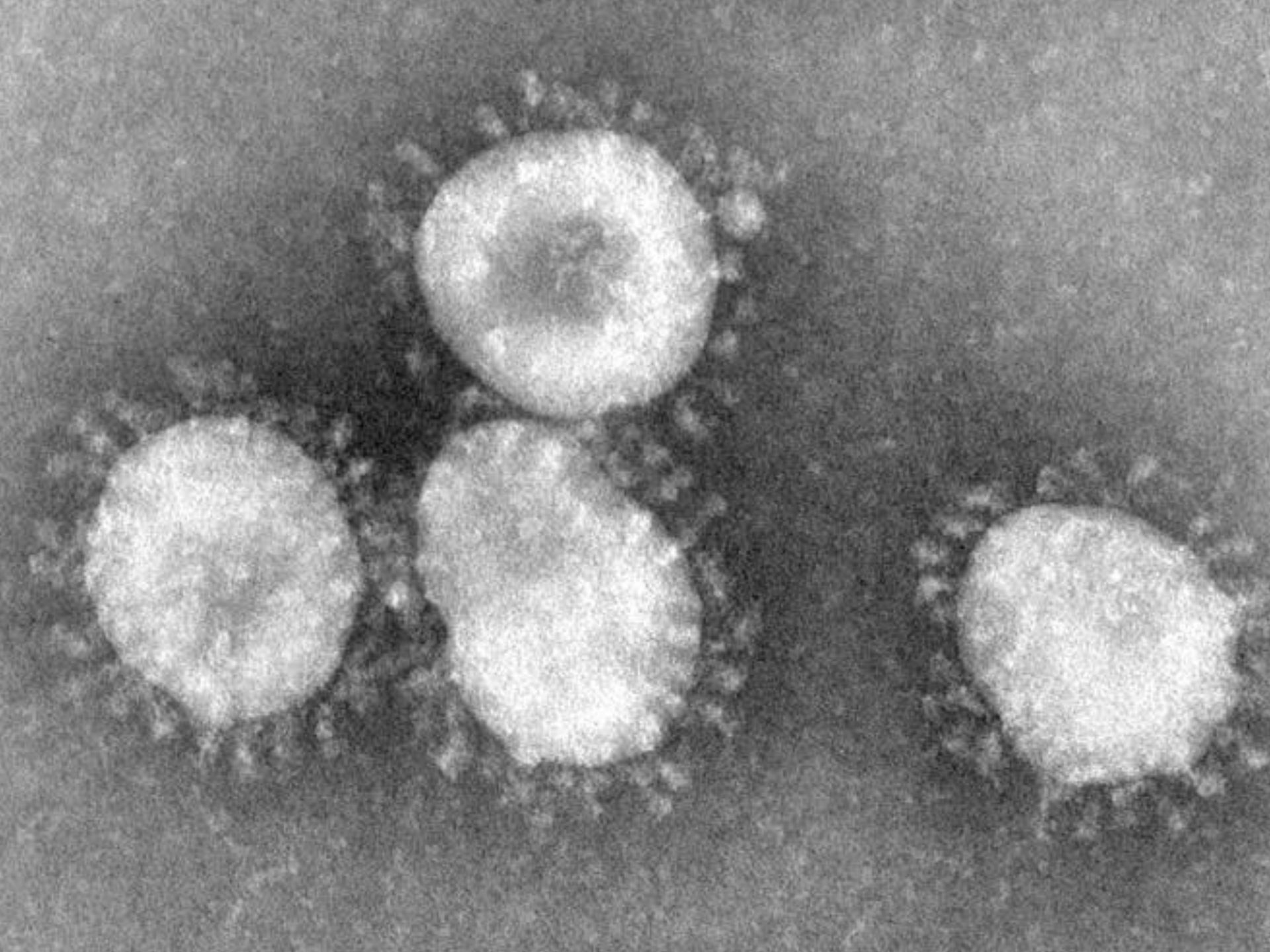
Bay of
Bengal

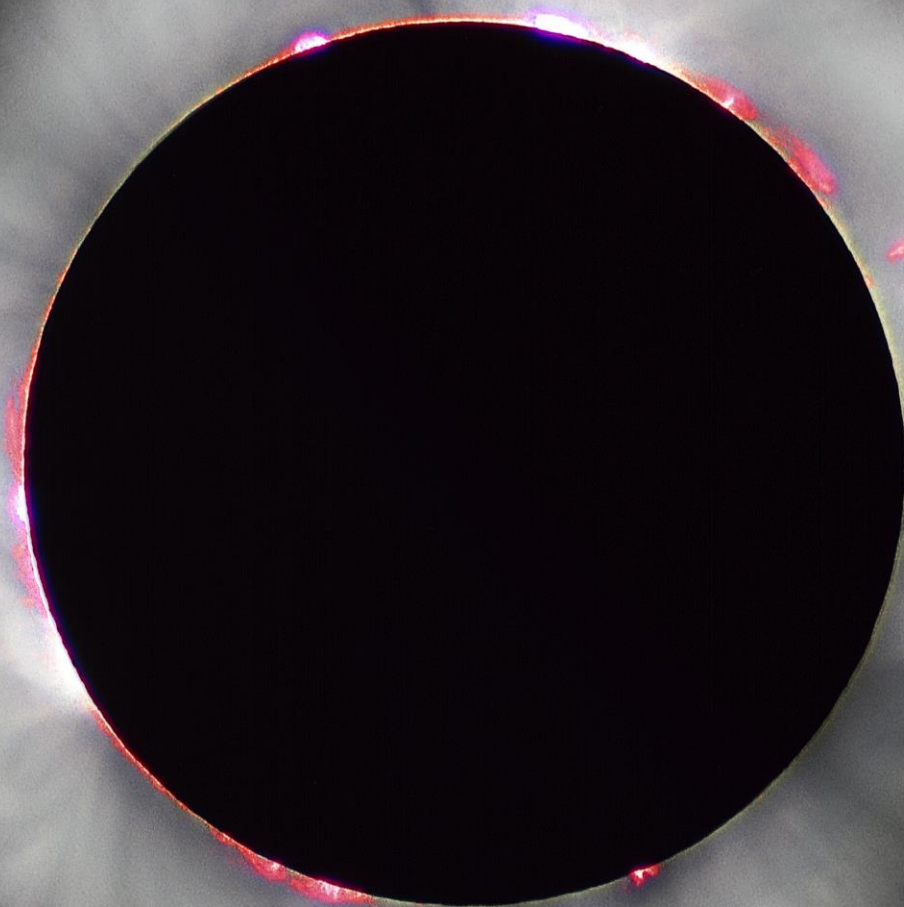
Philippine
Sea

South
China
Sea

Indian

Širenje
SARS-a





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28321	ccucgaggcc	agggcguucc	aaucaacacc	aauagugguc	cagaugacca	aauuggcuac
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29161	aaagacaacg	ucauacugcu	gaacaagcac	auugacgcau	acaaaacauu	cccaccaaca
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29281	aagaagcagc	ccacugugac	ucuucuuccu	gcggcugaca	uggaugauuu	cuccagacaa
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29401	accacacaag	gcagaugggc	uauguaaacg	uuuucgcaau	uccguuuacg	auacauaguc
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28441	agaugguacu	ucuauuaccu	aggaacuggc	ccagaagcuu	cacuucccua	cggcgcuaac
28501	aaagaaggca	ucguaugggu	ugcaacugag	ggagccuuga	auac C cccaa	agaccacauu
28561	ggcaccgca	auccuaauaa	caaugcugcc	accgugcuac	aacuuccuca	aggaacaaca
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29221	ga U ccuaaaa	aggacaaaaa	gaaaaagacu	gaugaagcuc	agccuuugcc	gcagagacaa
29281	aagaagcagc	ccacugugac	ucuucuuccu	gcggcugaca	uggaugauuu	cuccagacaa
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29641	cugccuauau	ggaagagccc	uaauguguaa	aauuuuuuuu	aguagugcua	uccccaugug
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28501	aaagaaggca	ucguau U ggg	ugcaacugag	ggagccuuga	auac C cccaa	agaccacauu
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28621	uugccaaaag	gcuuc A acgc	agaggggaagc	agagggcggca	gucaagccuc	uucucgcucc
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28801	uugaaccagc	uugagagcaa	aguuucuggu	aaaggccaac	aacaacaagg	cca G acuguc
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28741	C G ugcucgaa	uggcuagcgg	agguggugaa	acugcccucg	cgcuaauugcu	gcuagacaga
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29581	cauu A ucauc	gaggccacgc	ggaguacgau	cgaggguaca	gugaauaaug	cua A ggagag
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29701	auuuuaauag	cuucuuagga	gaugacaaa	aaaaaaaaa	aaaaaaaaa	a



Pitanja u vezi sa SARS-om

- Koja životinja nam je donela SARS?

Pitanja u vezi sa SARS-om

- Koja životinja nam je donela SARS?
- Kako smo se prvobitno zarazili?

Pitanja u vezi sa SARS-om

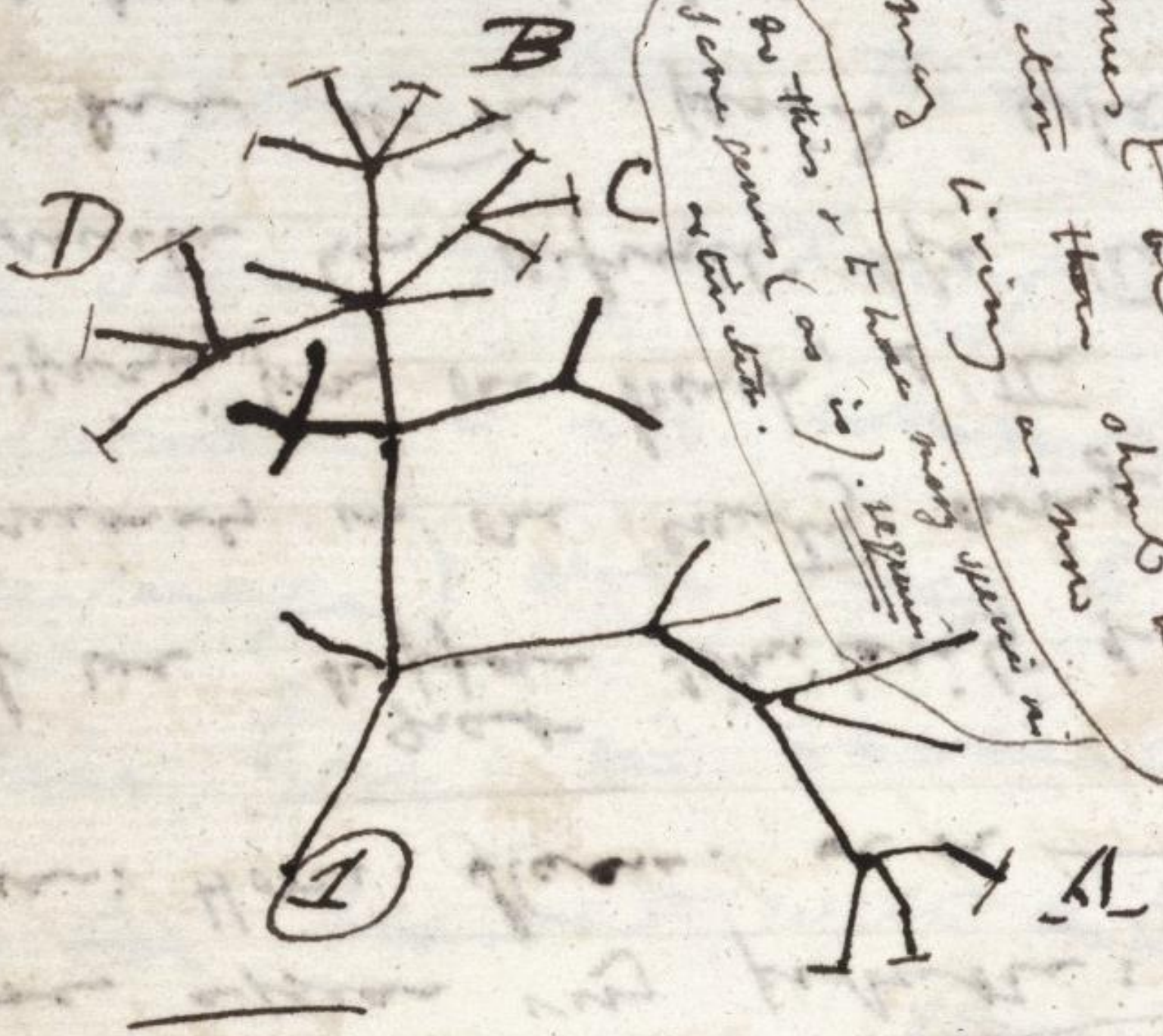
- Koja životinja nam je donela SARS?
- Kako smo se prvobitno zarazili?
- Kako se SARS širio po svetu?

Pitanja u vezi sa SARS-om

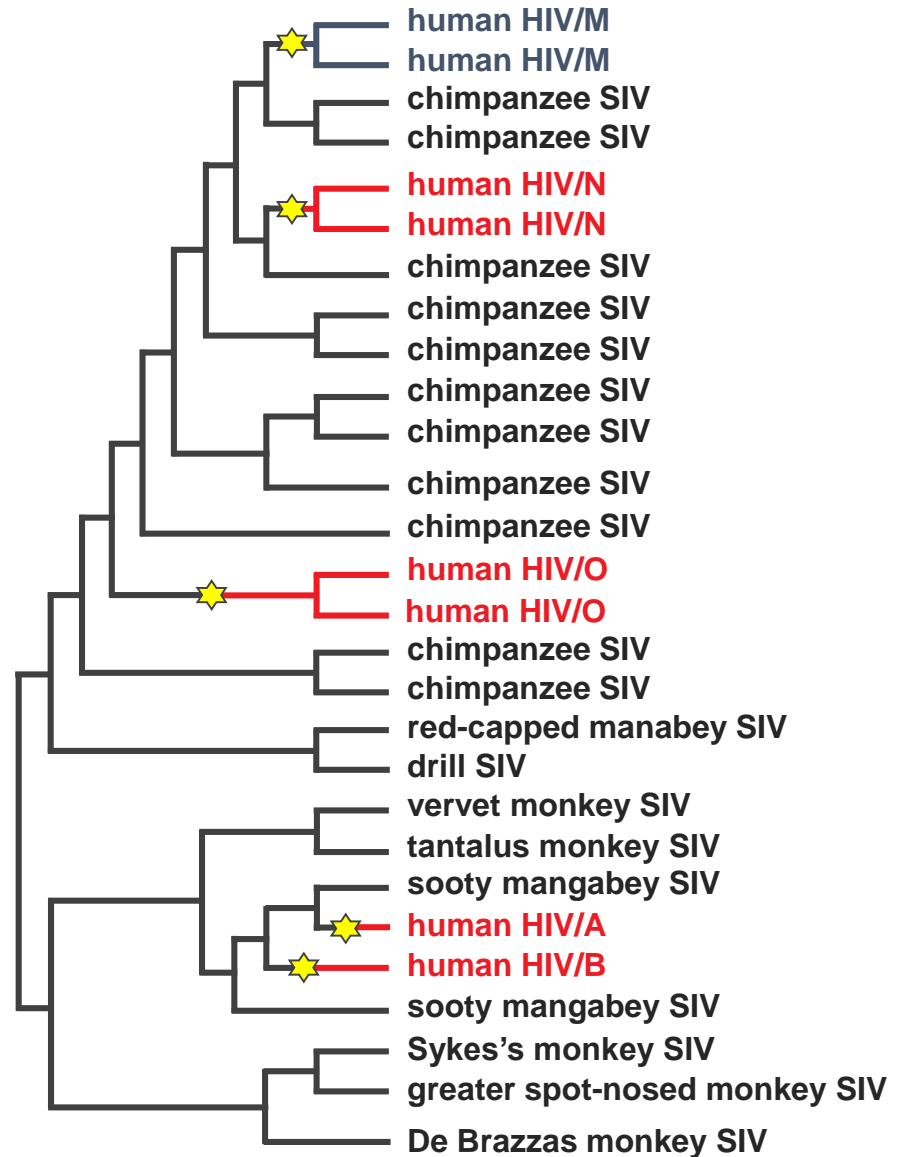
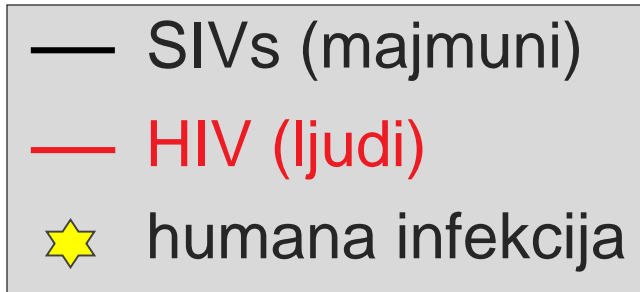
- Koja životinja nam je donela SARS?
- Kako smo se prvobitno zarazili?
- Kako se SARS širio po svetu?
- Sva ova pitanja spadaju u domen filogenetske analize koja se bavi rekonstrukcijom evolutivnih stabala

must be the other one
 or than than or more
 living species in
 more

of the this & E have many species
 of some persons (as is) repeated
 as the text.



Evolutivno stablo za HIV



Pregled

- Izbijanje epidemije
- **Transformacija matrice rastojanja u evolutivno stablo**
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- Problem male parsimonije
- Problem velike parsimonije

Konstrukcija matrice rastojanja

Vrsta Poravnanje

Šimpanza ACGTAGGCCT

Čovek ATGTAAGACT

Foka TCGAGAGCAC

Kit TCGAAAGCAT

Konstrukcija matrice rastojanja

$D_{i,j}$ = broj različitih simbola u i -tom i j -tom redu višestrukog poravnanja

Vrsta	Poravnanje	Matrica rastojanja			
		Šimpanza	Čovek	Foka	Kit
Šimpanza	ACGTAGGCCT	0	3	6	4
Čovek	ATGTAAGACT	3	0	7	5
Foka	TCGAGAGCAC	6	7	0	2
Kit	TCGAAAGCAT	4	5	2	0

Konstrukcija matrice rastojanja

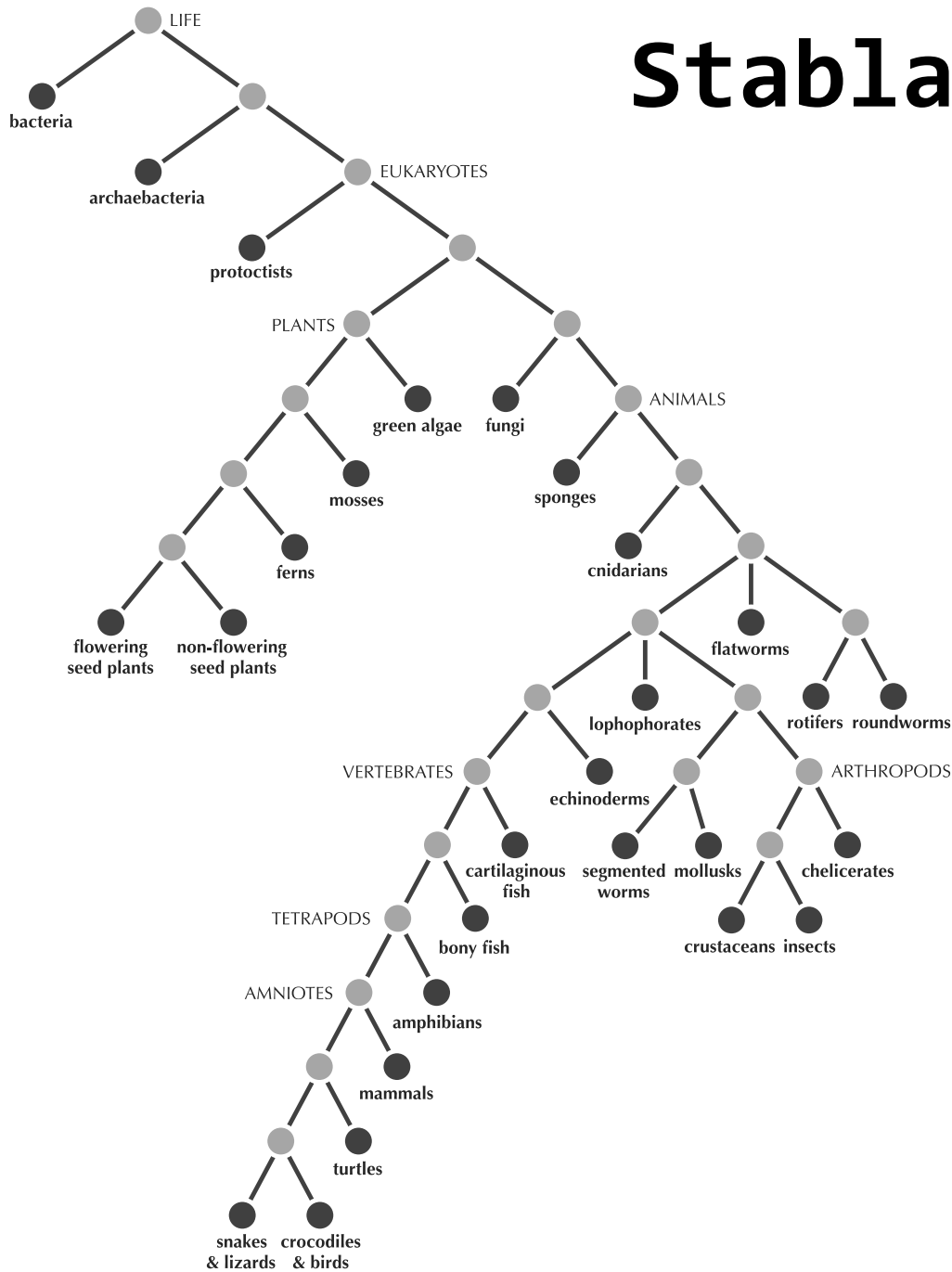
$D_{i,j}$ = broj različitih simbola u i -tom i j -tom redu višestrukog poravnanja

Vrsta	Poravnanje	Matrica rastojanja			
		Šimpanza	Čovek	Foka	Kit
Šimpanza	ACGTAGGCCT	0	3	6	4
Čovek	ATGTAAAGACT	3	0	7	5
Foka	TCGAGAGCAC	6	7	0	2
Kit	TCGAAAGCAT	4	5	2	0

Spike protein

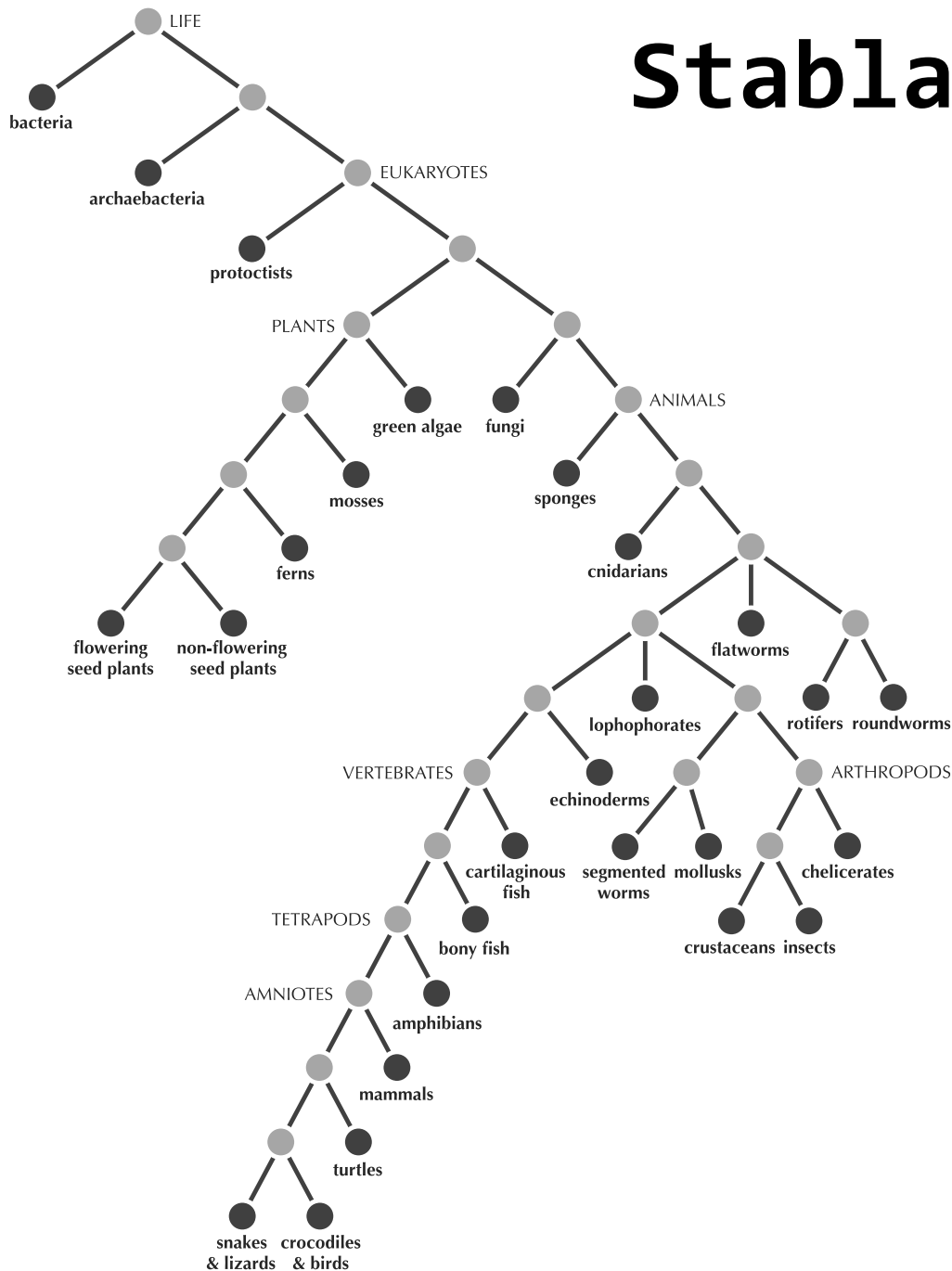
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121 tnvviracnf elcdnpffav skpmgtqtht mifdnafnct feyisdafsl dvseksgnfk
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1021 rvdhfcgkgyh lmsfpqaaph gvvflhvtyv psqernftta paichegkay fpregvfvfn
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Stabla



Stablo: povezani aciklički graf.

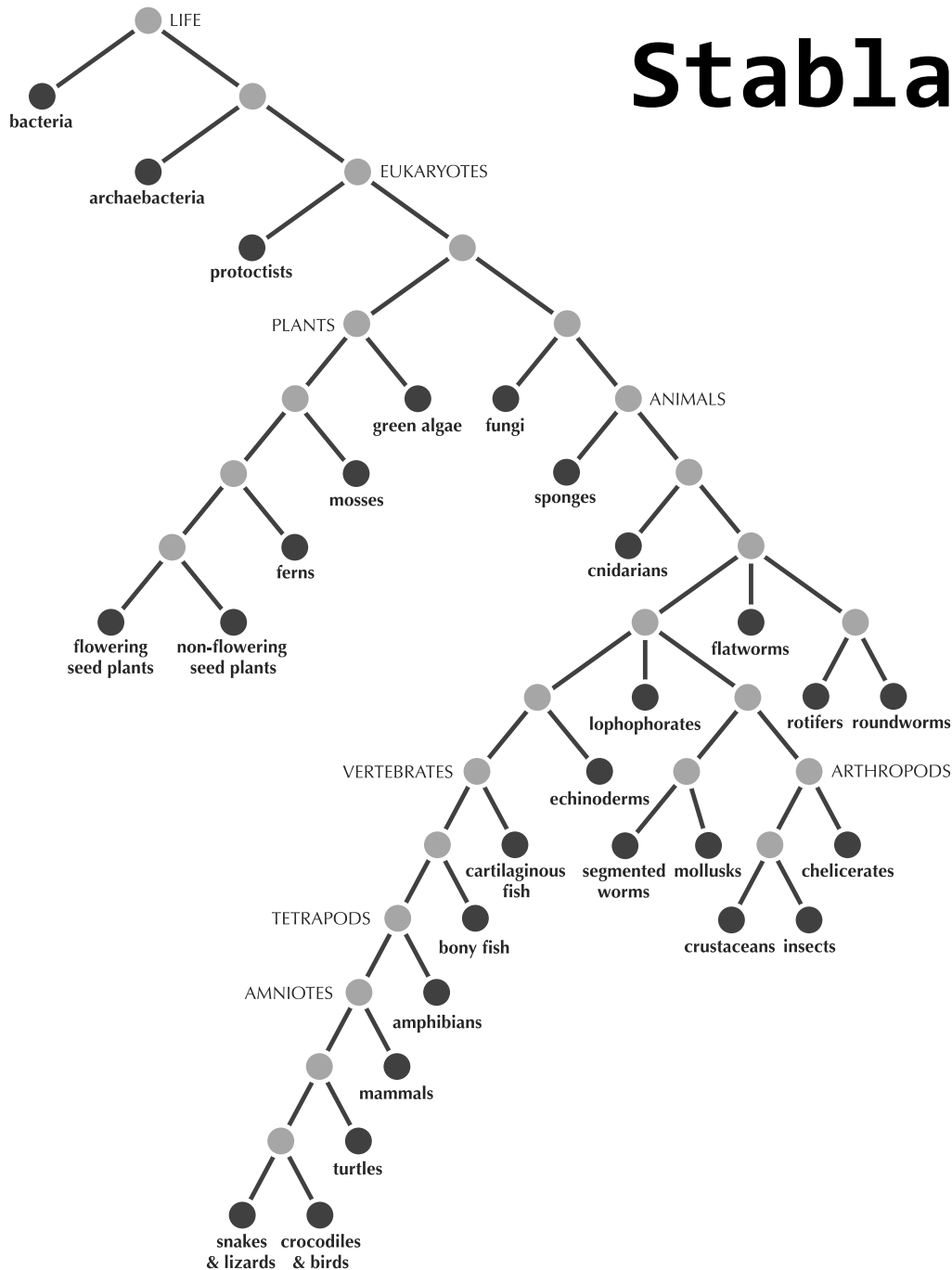
Stabla



Stablo: povezani aciklički graf.

Listovi (*degree=1*): današnje vrste

Stabla

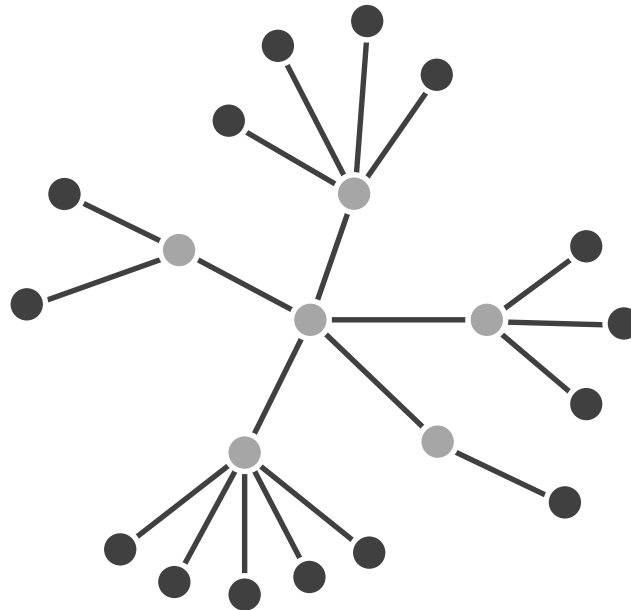
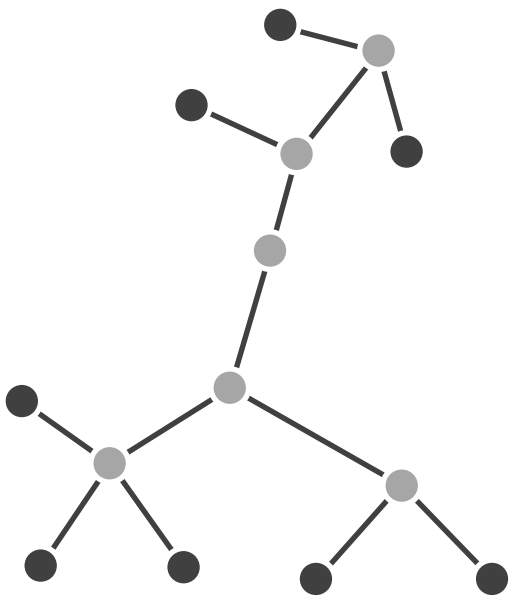


Stablo: povezani aciklički graf.

Listovi ($degree=1$): današnje vrste

Unutrašnji čvorovi ($degree \geq 1$): izumrle vrste

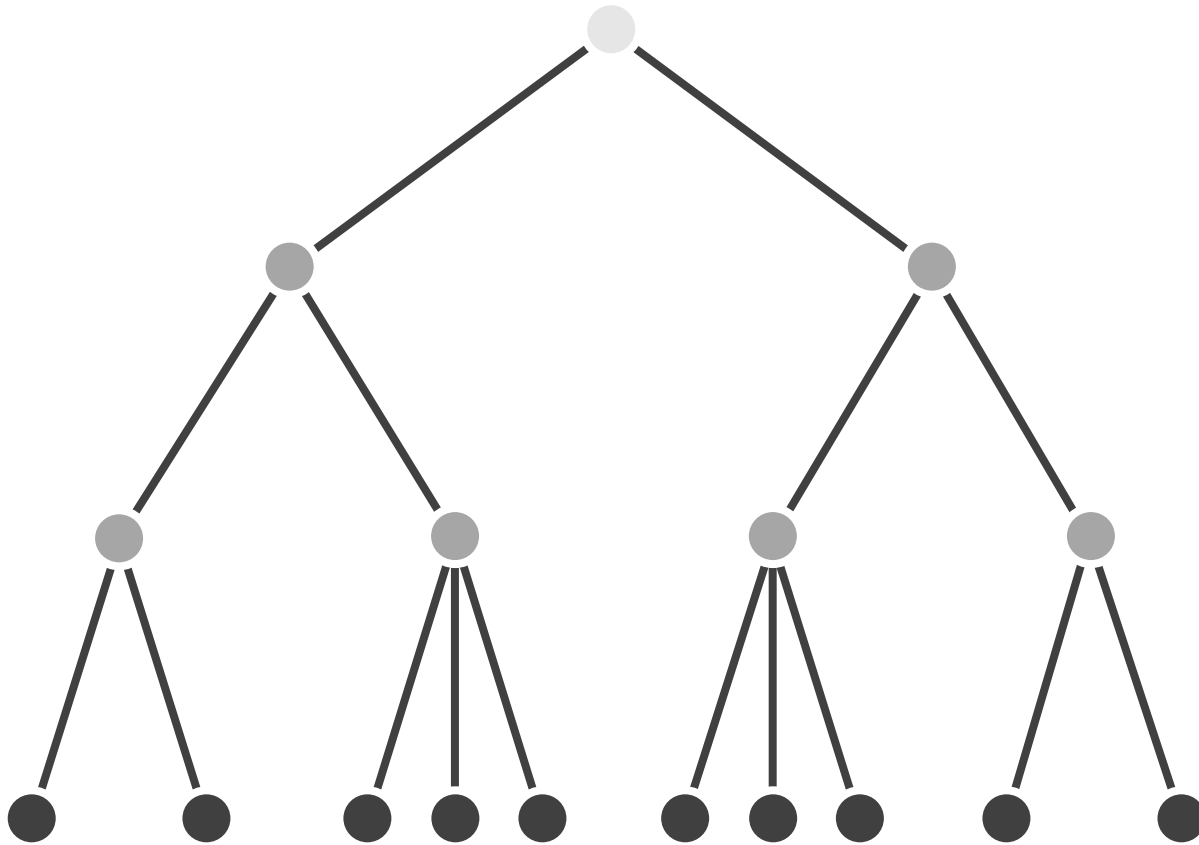
Stabla



Za povezana aciklična stabla se može pokazati:

1. Svako stablo sa bar dva čvora sadrži bar dva lista.
2. Svako stablo sa n čvorova sadrži tačno $n - 1$ grana.

Stabla



Najdalji predak



vreme

Danas

Koreni čvor: najdalji zajednički predak

Filogeneza na osnovu rastojanja

Problem filogeneze na osnovu rastojanja:
Konstruisati evolutivno stablo na osnovu
matrice rastojanja.

- **Ulaz:** Matrica rastojanja.
- **Izlaz:** Nekoreno stablo koje *odgovara* matrici rastojanja.

Ovo nije dobro
definisan
problem!

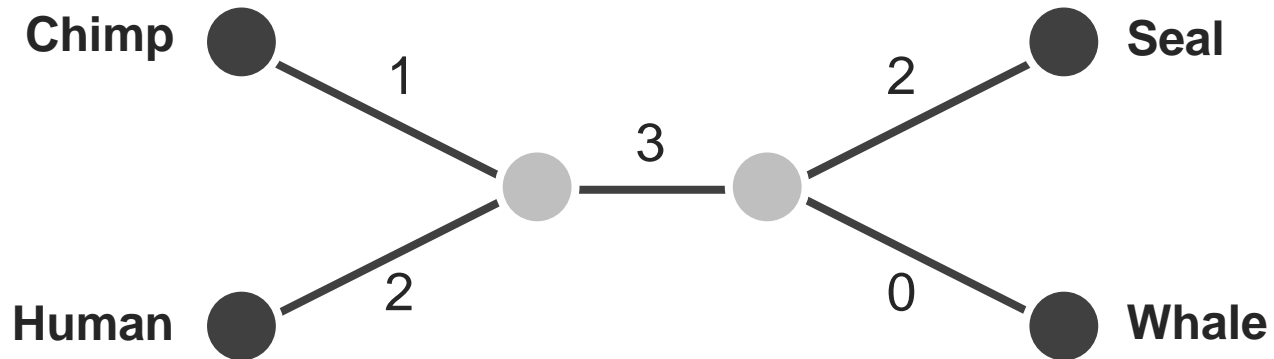


Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

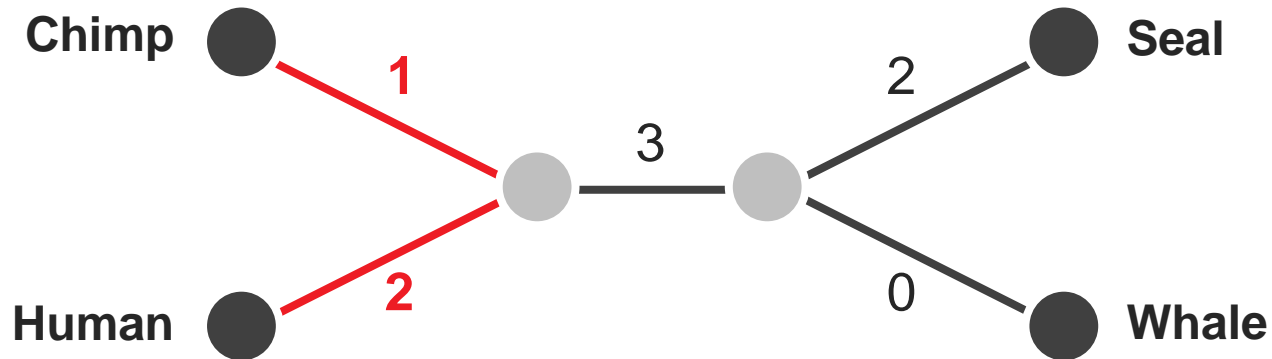
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



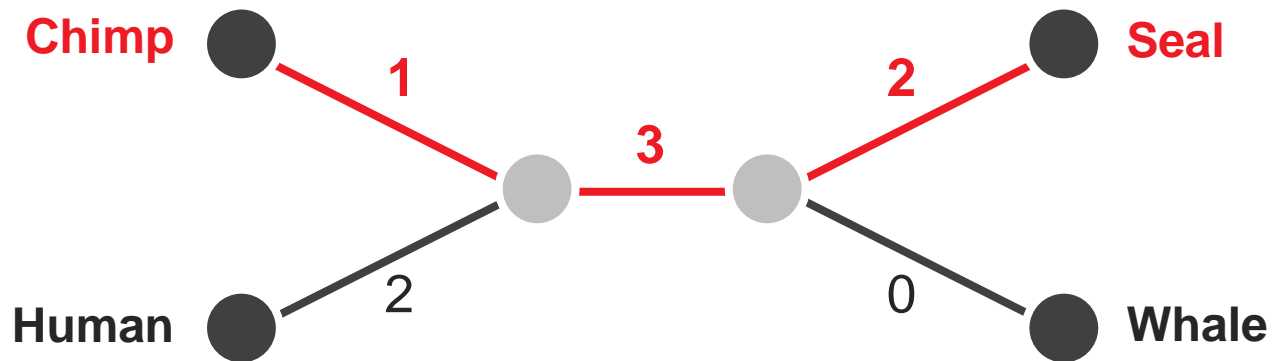
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



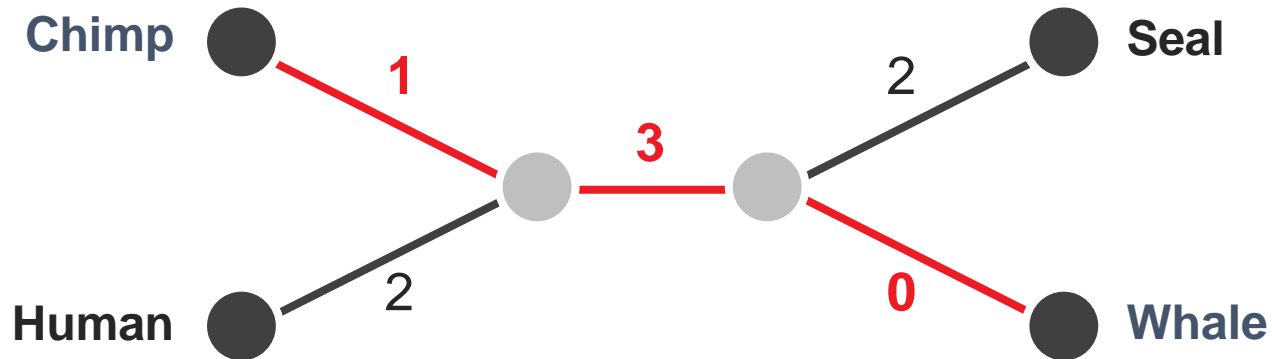
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



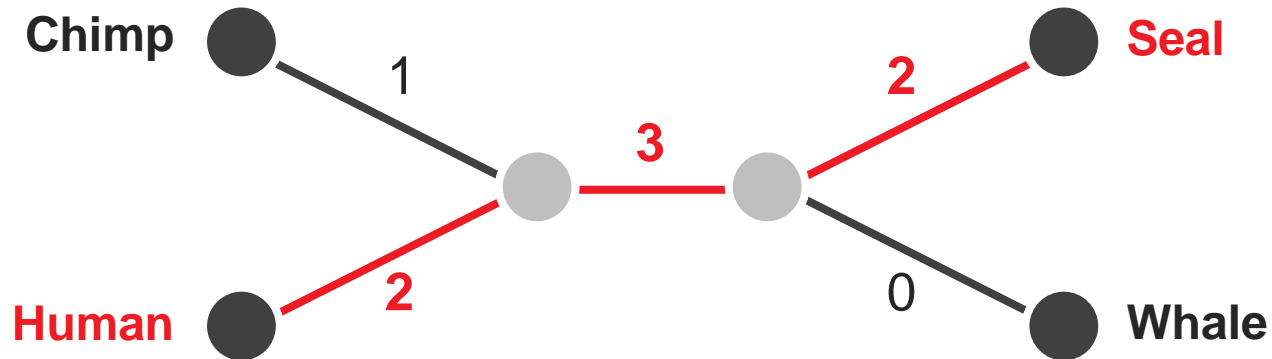
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



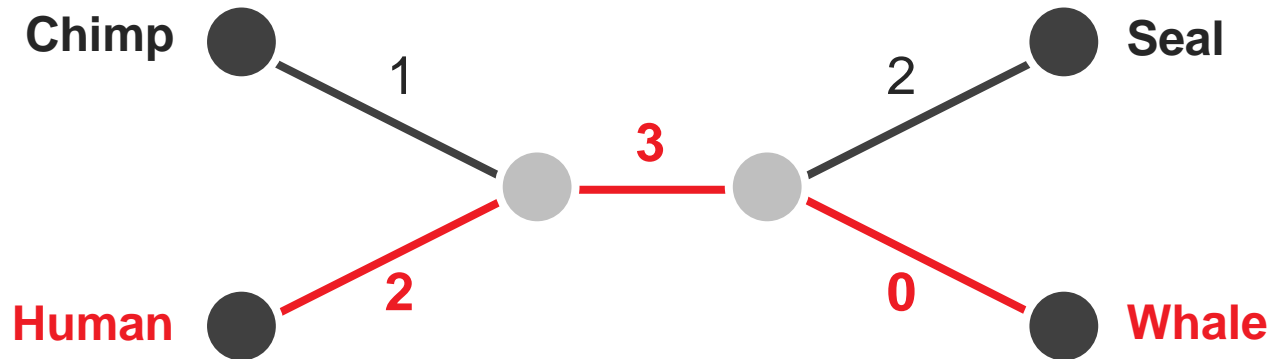
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



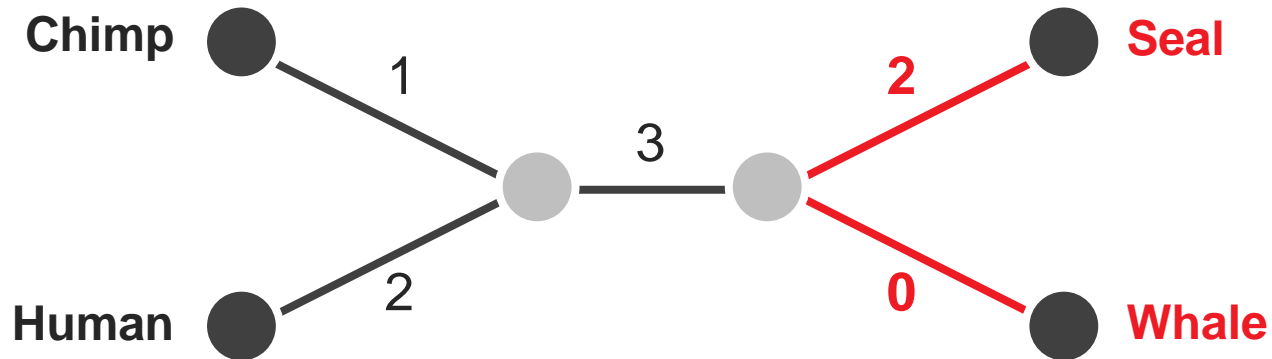
Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



Kada stablo odgovara matrici?

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



Fiłogeneza na osnovu rastojanja

Pokušajmo da konstruišemo stablo koje odgovara sledećoj matrici:

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

Nijedno odgovarajuće stablo

Pokušajmo da konstruišemo stablo koje odgovara sledećoj matrici:

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

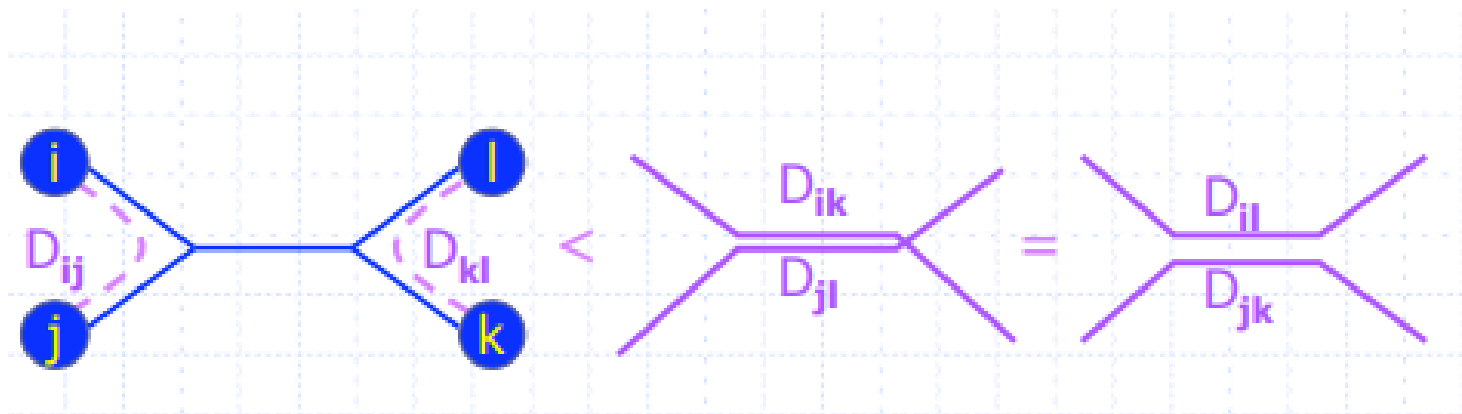
Da bi za datu matricu bilo moguće konstruisati odgovarajuće evolutivno stablo, matrica mora biti *aditivna* (u nastavku)

Aditivne matrice

- Definicija: Za dato evolutivno stablo, matricu koja opisuje rastojanja između njegovih listova zovemo *aditivnom matricom*
- Teorema: Matrica D je aditivna akko za proizvoljna 4 indeksa u matrici i, j i k, l važi:

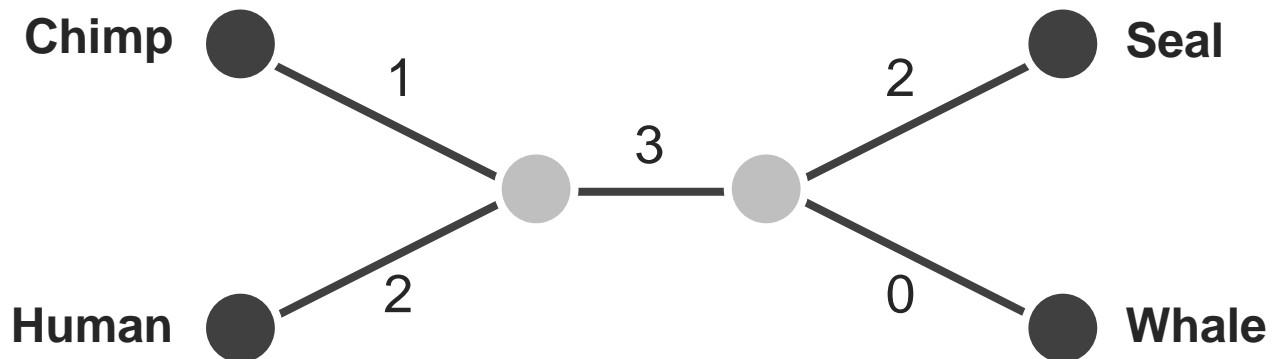
$$D_{ij} + D_{kl} \leq D_{ik} + D_{jl} = D_{il} + D_{jk}$$

- Ova teorema sugerije kako da na osnovu matrice povežemo listove u evolutivnom stablu:



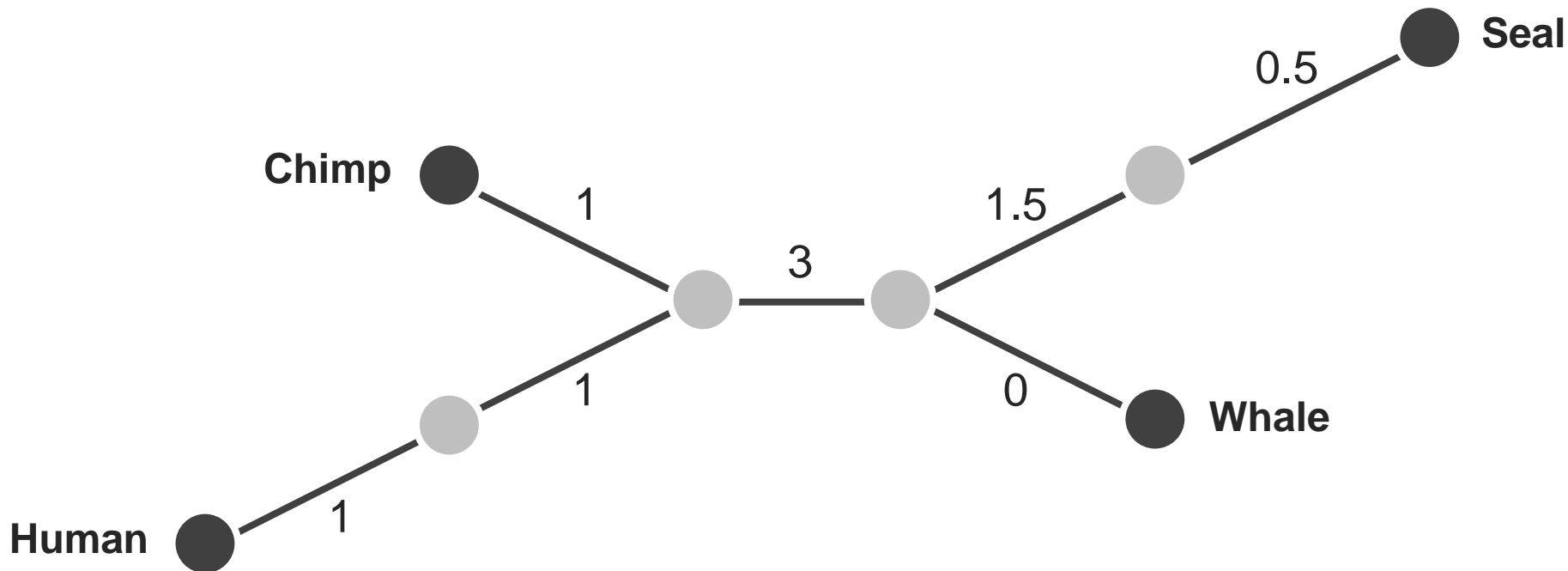
Više od jednog odgovarajućeg stabla

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

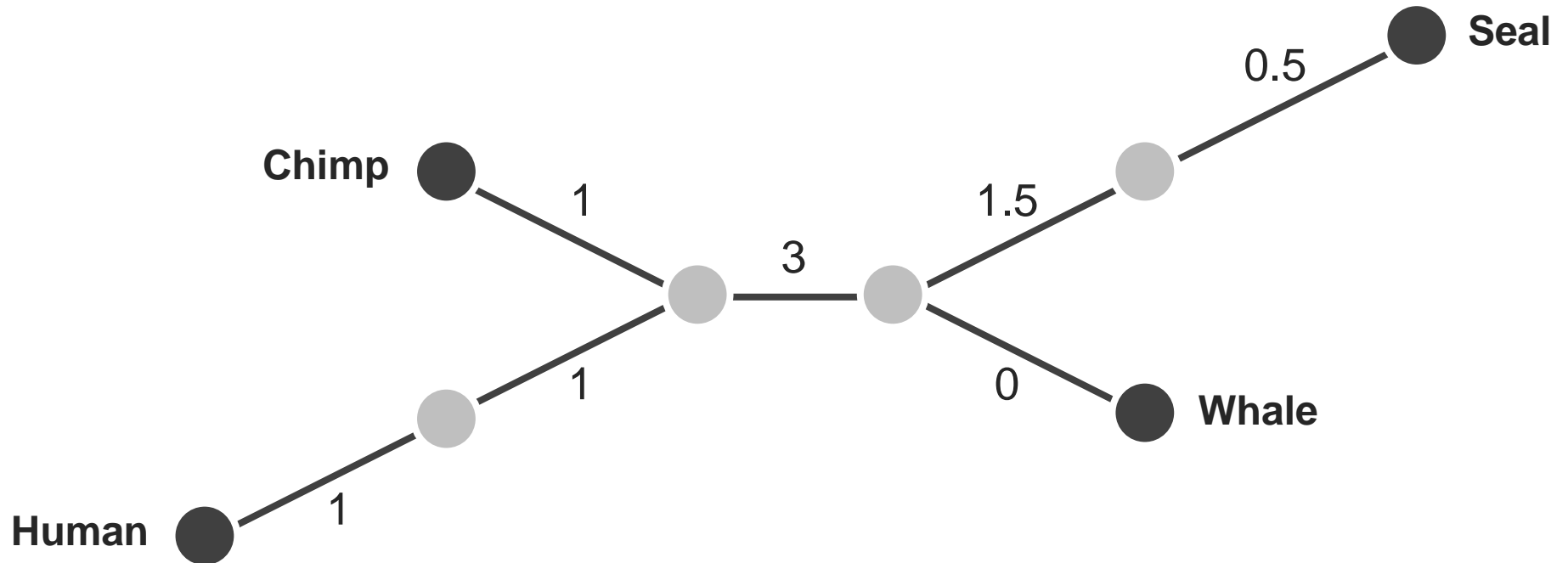
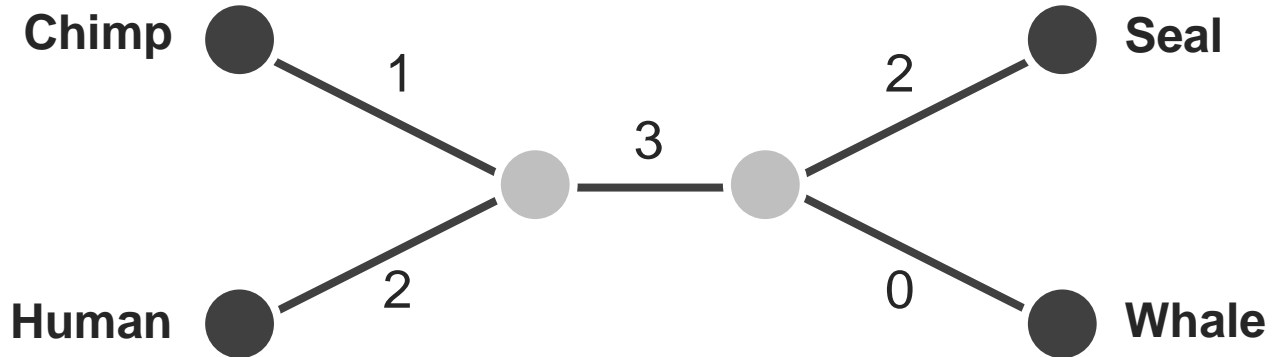


Više od jednog odgovarajućeg stabla

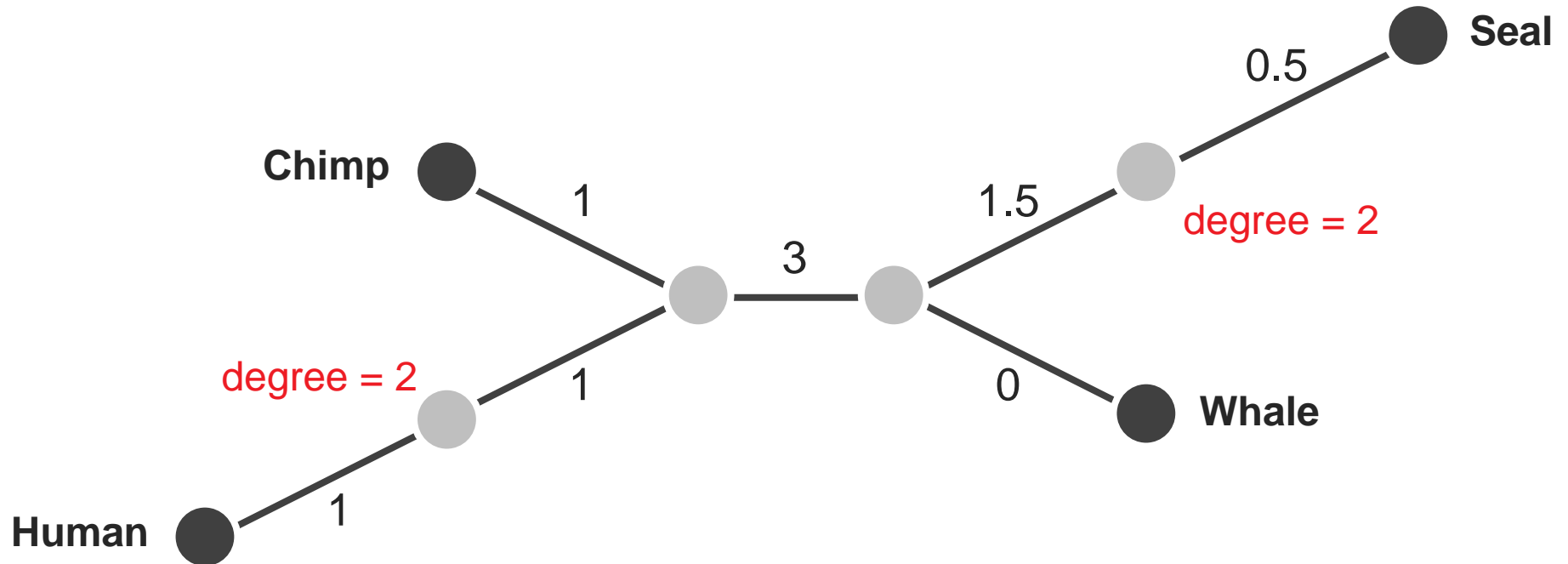
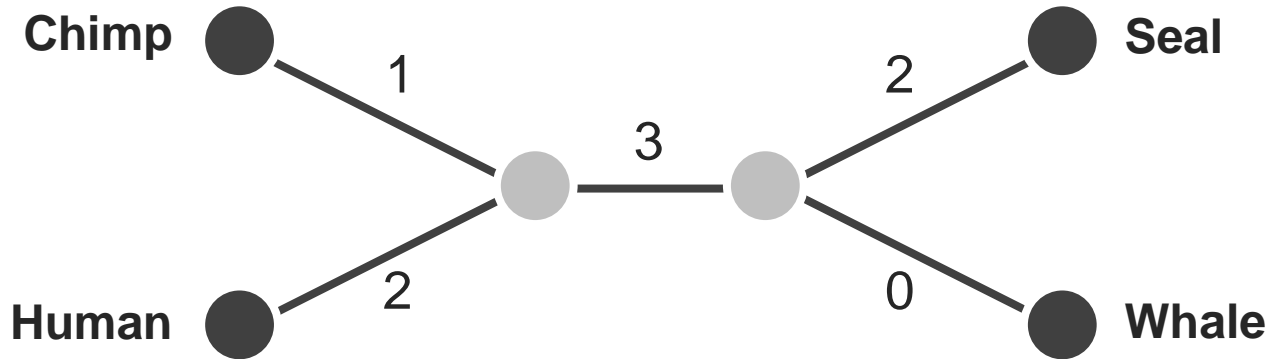
	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



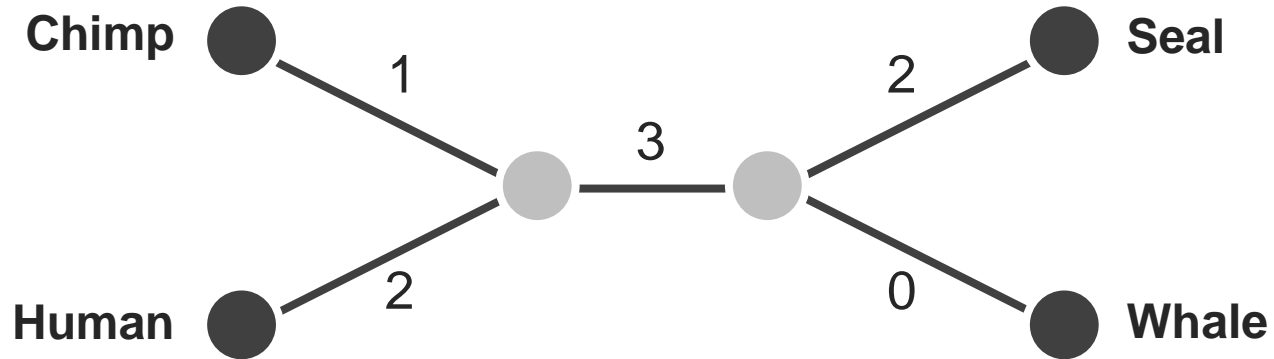
Koje stablo je bolje?



Koje stablo je bolje?

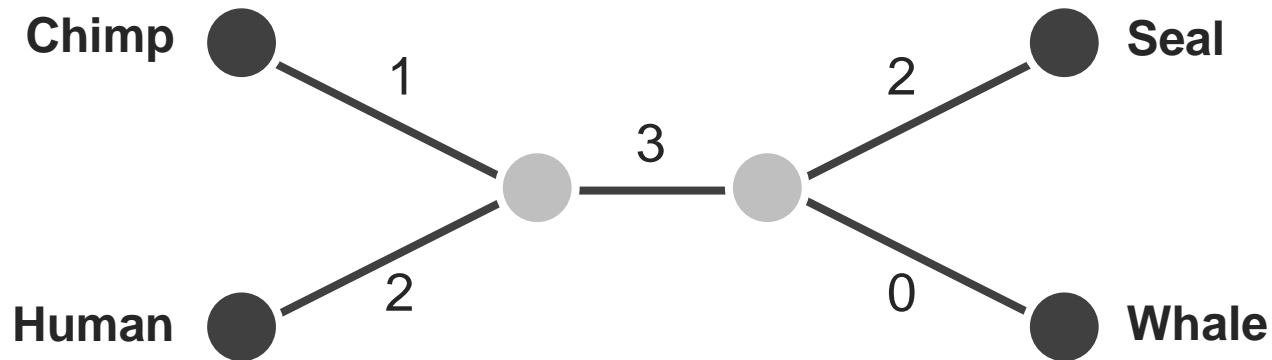


Koje stablo je bolje?



Prosto stablo: stablo bez čvorova stepena ≥ 2 .

Koje stablo je bolje?



Prosto stablo: stablo bez čvorova stepena =2.

Teorema: Postoji tačno jedno prosto stablo koje odgovara aditivnoj matrici

Reformulacija

Problem filogeneze na osnovu rastojanja:
Konstruisati evolutivno stablo na osnovu aditivne matrice rastojanja.

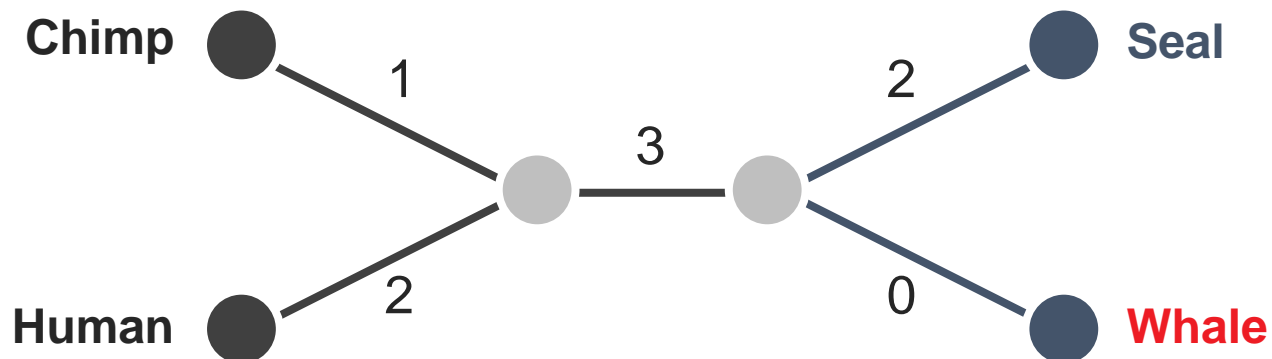
- **Ulaz:** Aditivna matrica rastojanja.
- **Izlaz:** Prosto stablo koje *odgovara* datoj matrici rastojanja.

Pregled

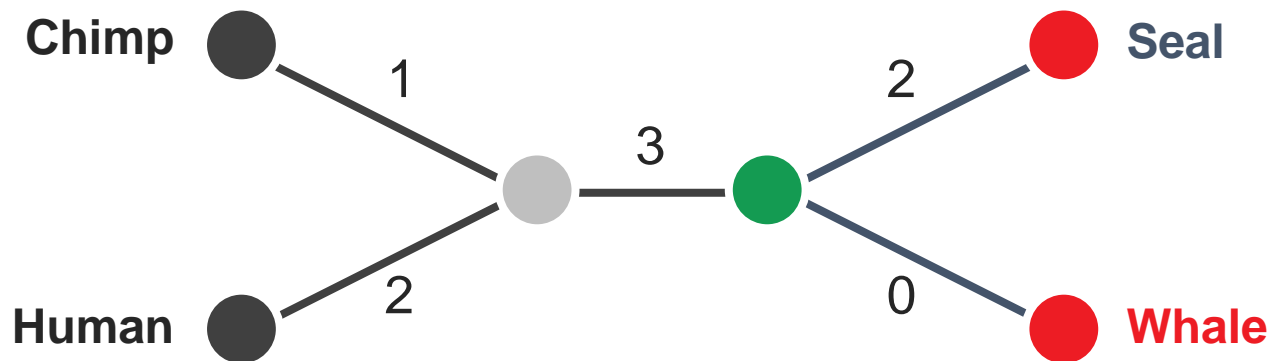
- Izbijanje epidemije
- Transformacija matrice rastojanja u evolutivno stablo
- **Prema algoritmu za rekonstrukciju filogenetskog stabla na osnovu rastojanja**
- *AdditivePhylogeny* algoritam
- Metod najmanjih kvadrata
- Ultrametrična evolutivna stabla
- Neighbour-Joining algoritam
- Rekonstrukcija stabla na osnovu karakteristika
- Problem male parsimonije
- Problem velike parsimonije

- Primetimo da minimalna pozitivna vrednost matrice rastojanja odgovara listovima u stablu koje povezuje zajednički roditelj
- Takve listove zovemo **susednim** listovima

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

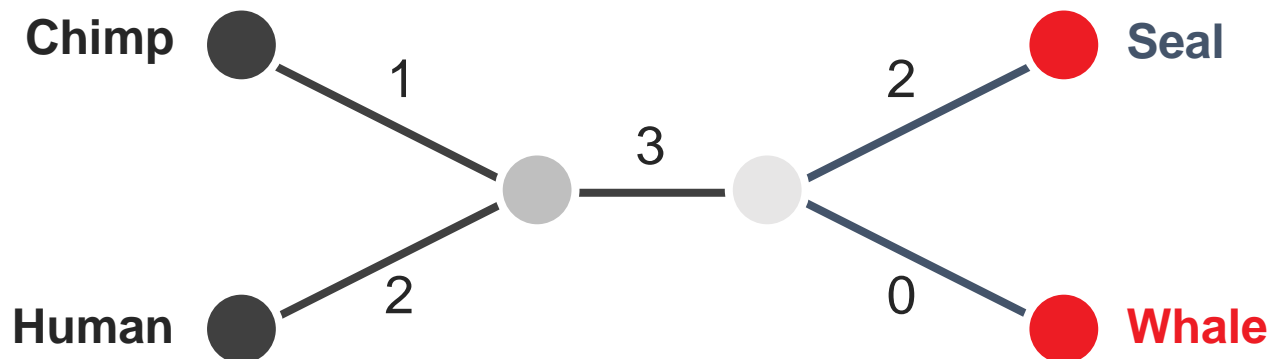


Foka i kit su **susedi** (dele isti roditeljski čvor).

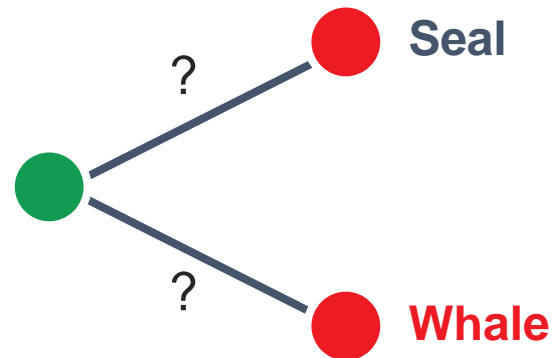


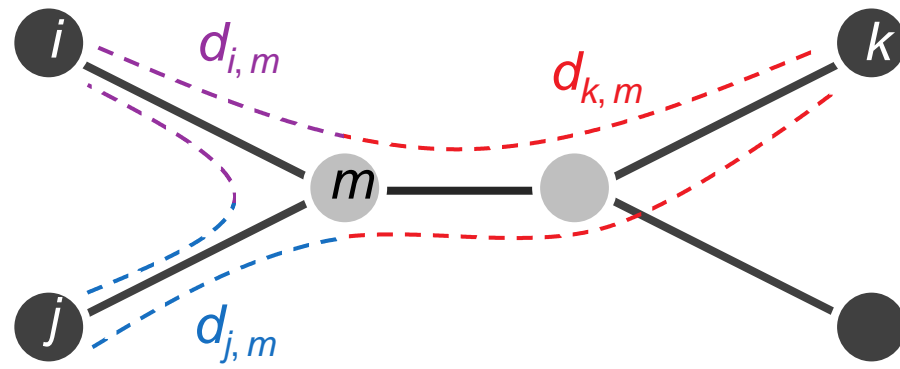
Foka i kit su **susedi** (dele isti roditeljski čvor).

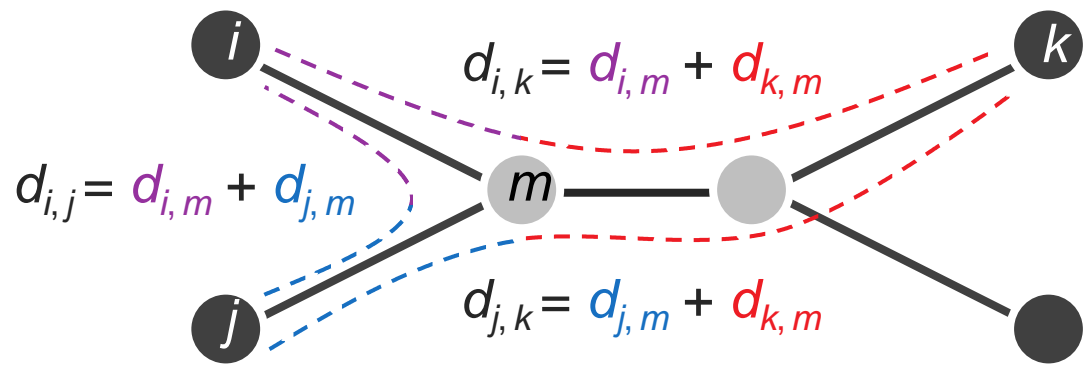
Teorema: Za svako prosto stablo sa bar četiri čvora postoji bar jedan par susednih listova.

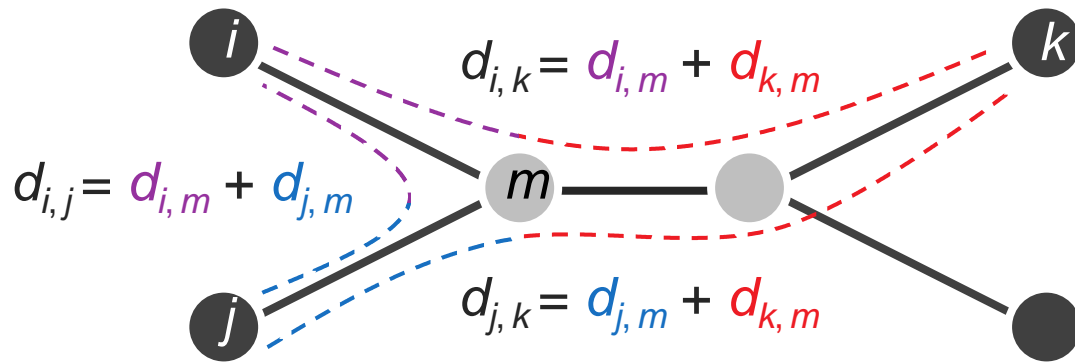


	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

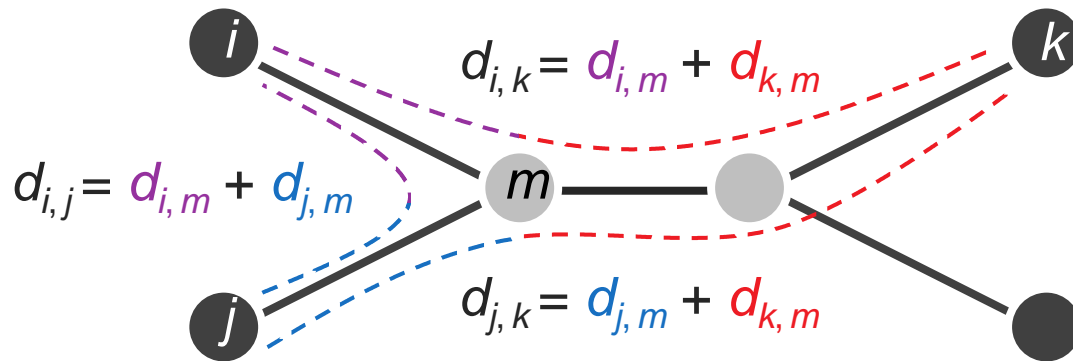








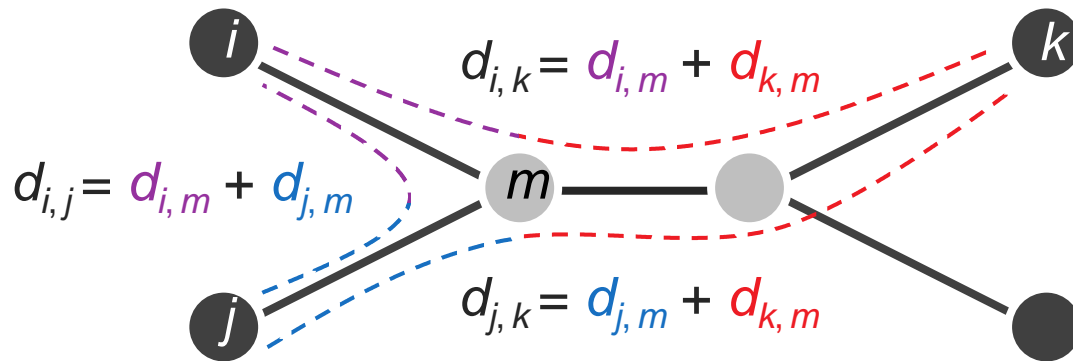
$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$



$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$d_{i,k}$, $d_{j,k}$, $d_{i,j}$ su rastojanja između listova koja su data u matrici rastojanja, dok npr rastojanja $d_{i,m}$ i $d_{j,m}$, koja predstavljaju rastojanja između lista i unutrašnjeg čvora, nisu data

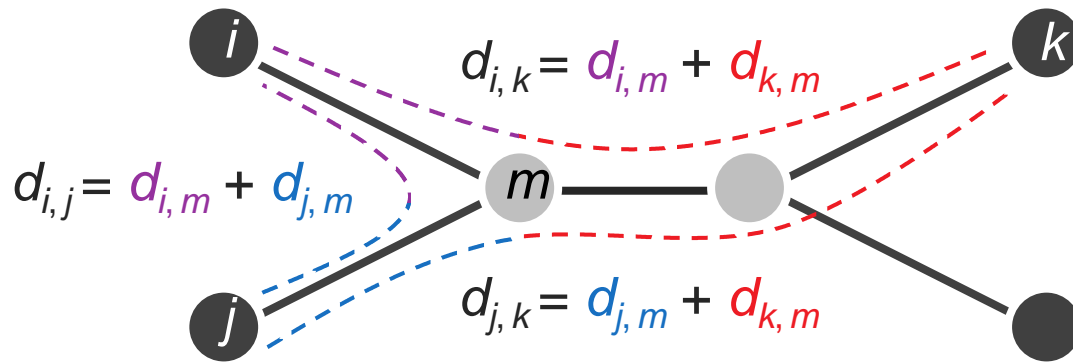


$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$$d_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

sa velikim D ($D_{i,k}$, $D_{j,k}$, $D_{i,j}$) označavamo elemente matrice rastojanja (rastojanje između dva lista u evolutivnom stablu), dok sa malim d označavamo rastojanje između bilo koja dva čvora u evolutivnom stablu

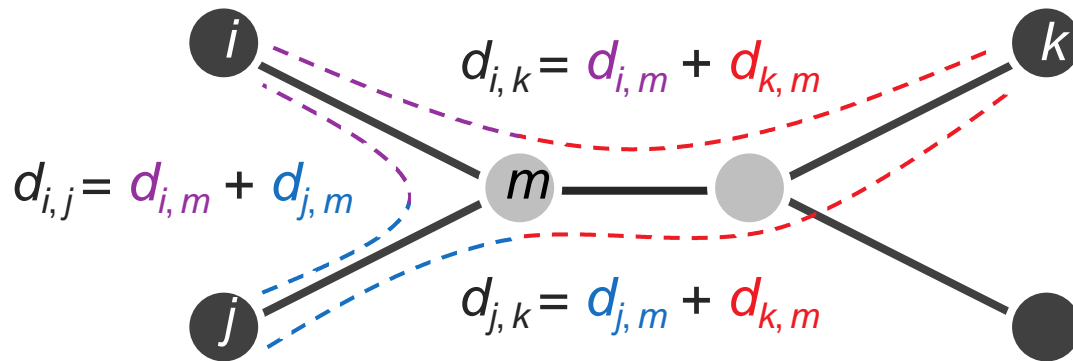


$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$$d_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

$$\therefore d_{i,m} = D_{i,k} - (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$



$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$$d_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

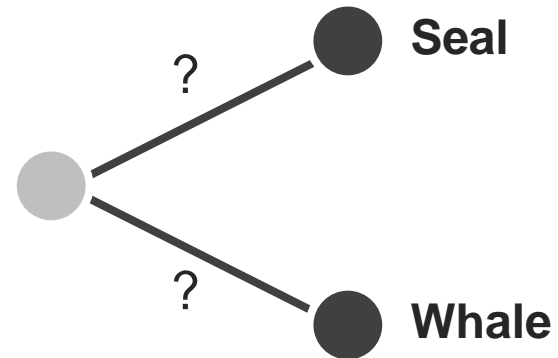
$$\therefore d_{i,m} = D_{i,k} - (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

$$d_{i,m} = (D_{i,k} + D_{i,j} - D_{j,k}) / 2$$

Analogno za drugog suseda
 $d_{j,m}$

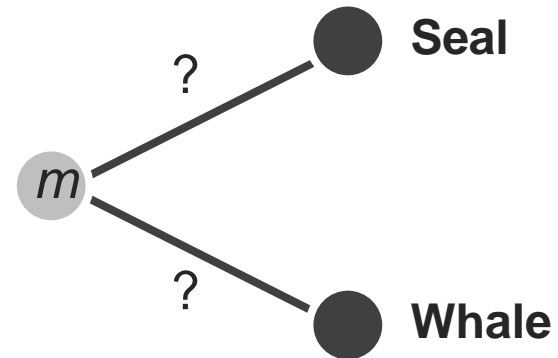
Obratimo pažnju da je čvor k proizvoljan – bilo koji list koji je različit od listova čija rastojanja do roditeljskog čvora tražimo

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



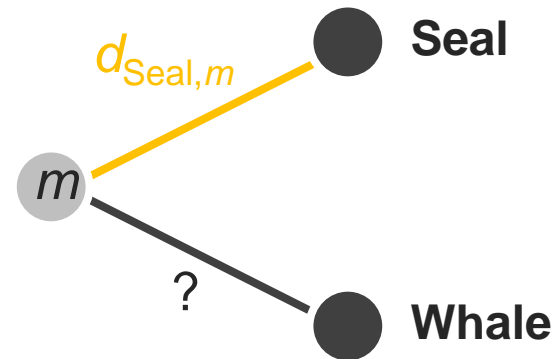
$$d_{i,m} = (D_{i,k} + D_{i,j} - D_{j,k}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



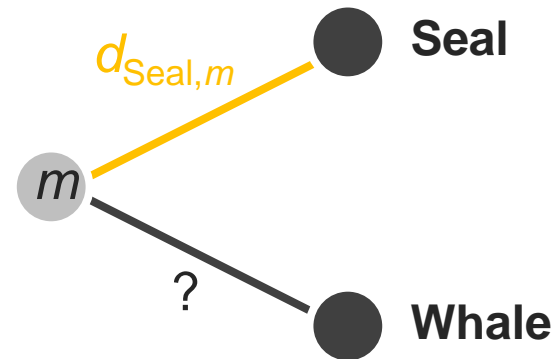
$$d_{i,m} = (D_{i,k} + D_{i,j} - D_{j,k}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



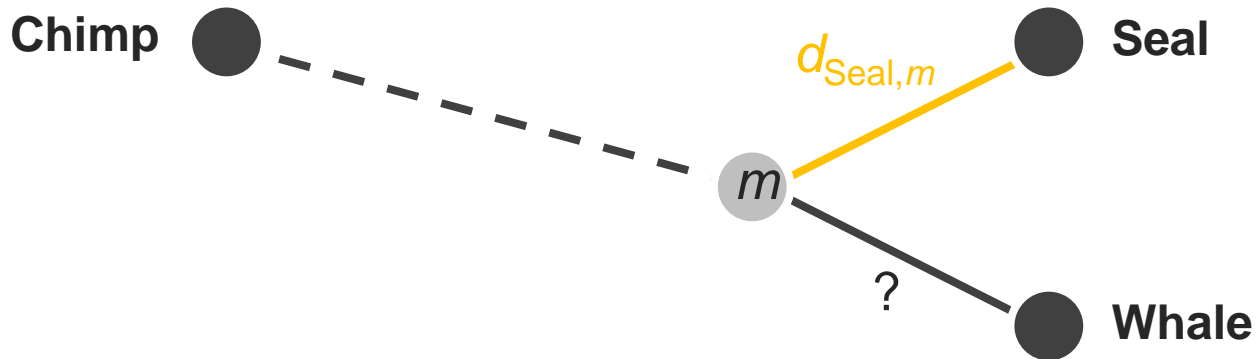
$$d_{\text{Seal},m} = (D_{\text{Seal},k} + D_{\text{Seal},j} - D_{j,k}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



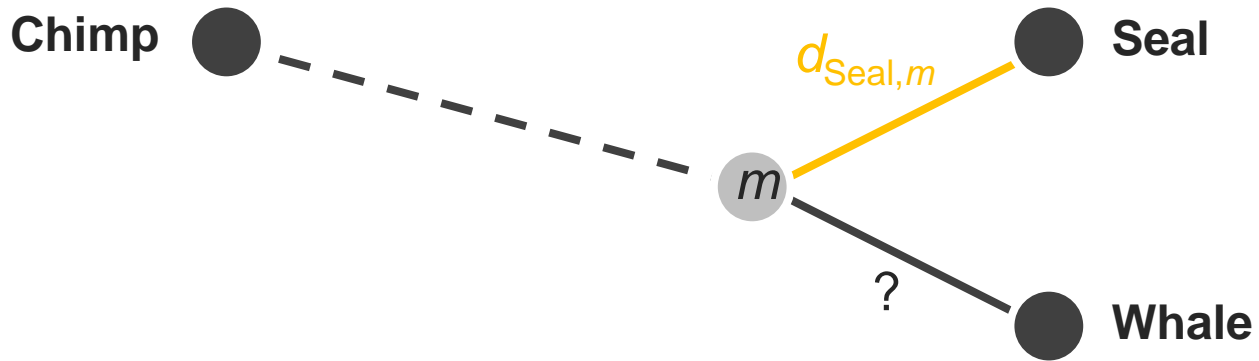
$$d_{\text{Seal},m} = (D_{\text{Seal},k} + D_{\text{Seal},\text{Whale}} - D_{\text{Whale},k}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



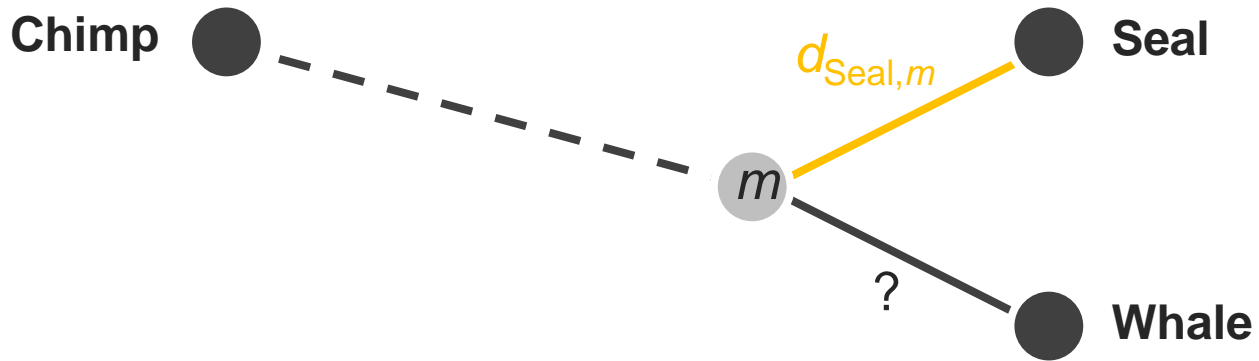
$$d_{Seal,m} = (D_{Seal,Chimp} + D_{Seal,Whale} - D_{Whale,Chimp}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



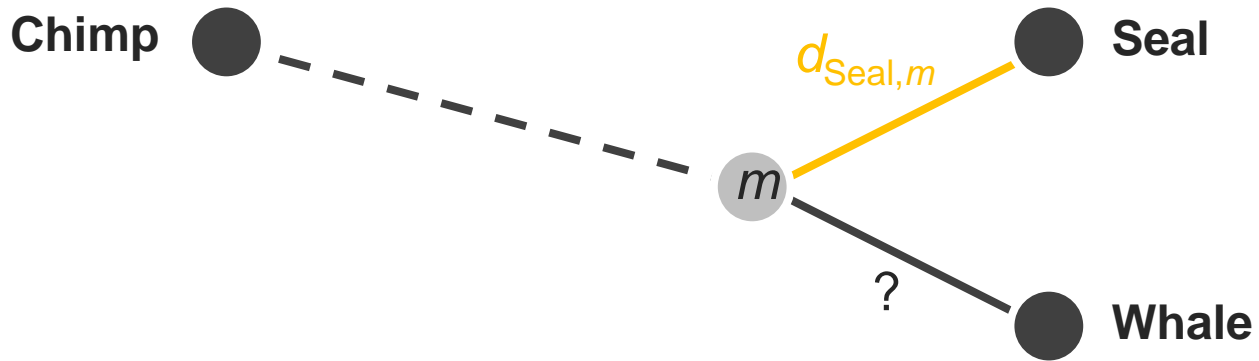
$$d_{Seal,m} = (6 + D_{Seal,Whale} - D_{Whale,Chimp}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



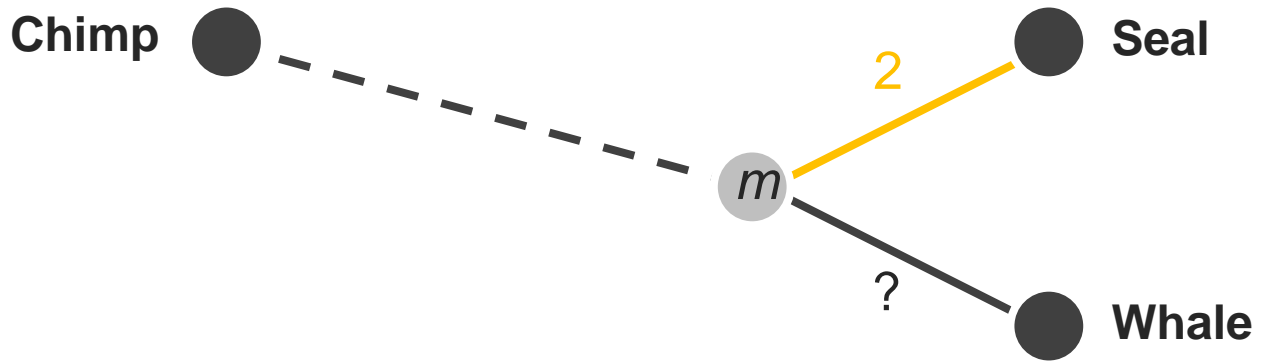
$$d_{Seal,m} = (6 + 2 - D_{Whale,Chimp}) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



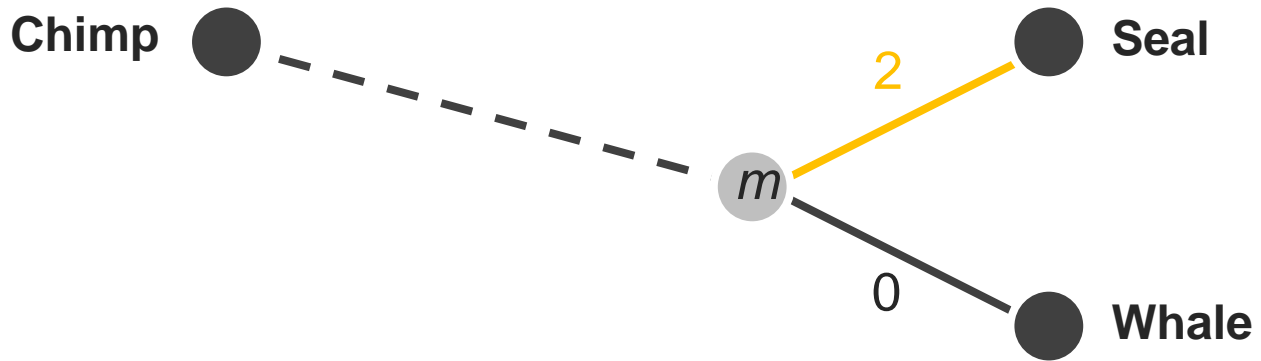
$$d_{Seal,m} = (6 + 2 - 4) / 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



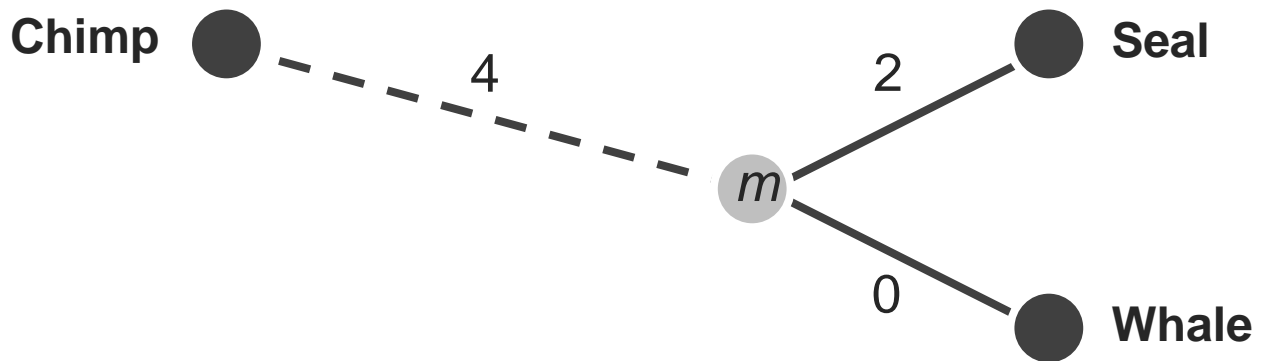
$$d_{\text{Seal},m} = 2$$

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

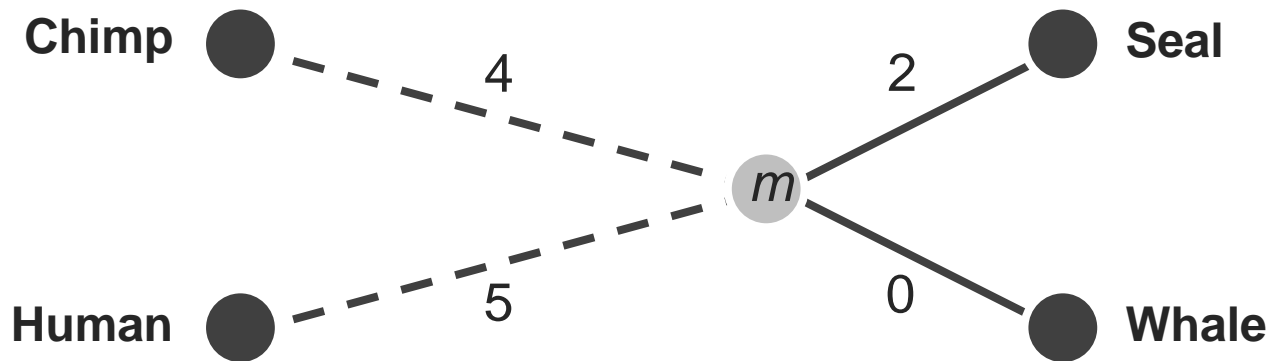


$$d_{\text{Seal},m} = 2$$

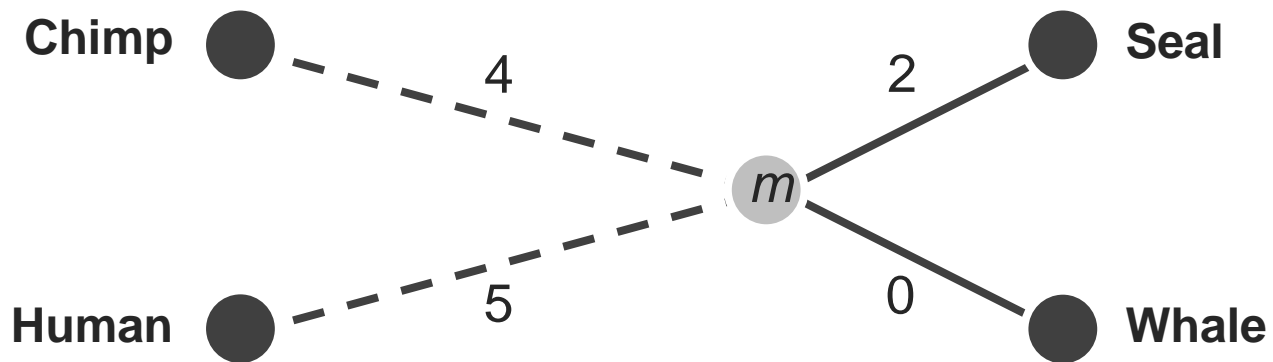
	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



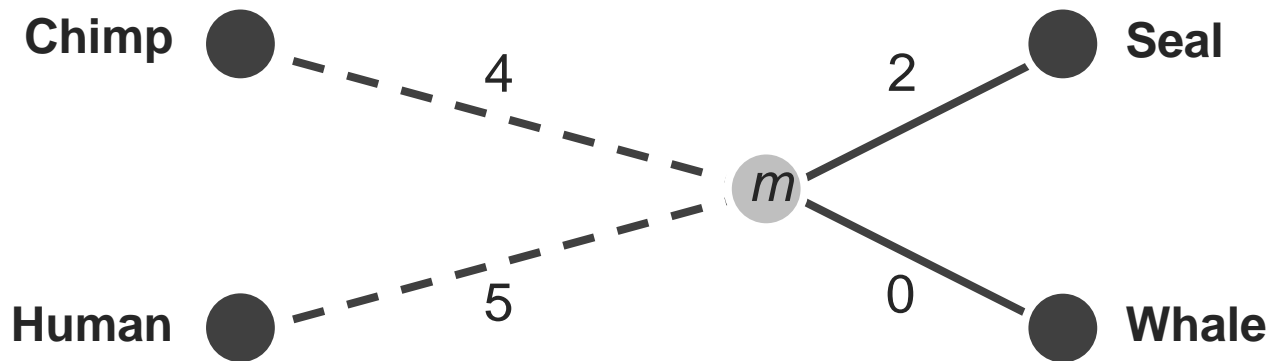
	Chimp	Human	Seal	Whale	<i>m</i>
Chimp	0	3	6	4	4
Human	3	0	7	5	5
Seal	6	7	0	2	2
Whale	4	5	2	0	0
<i>m</i>	4	5	2	0	0



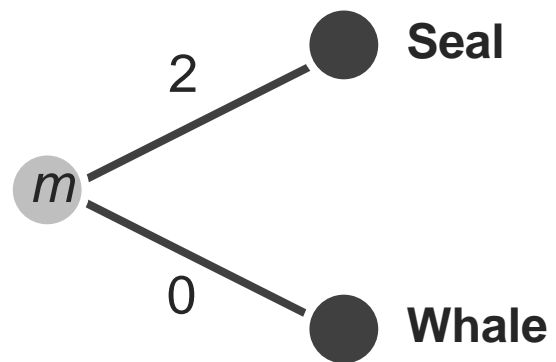
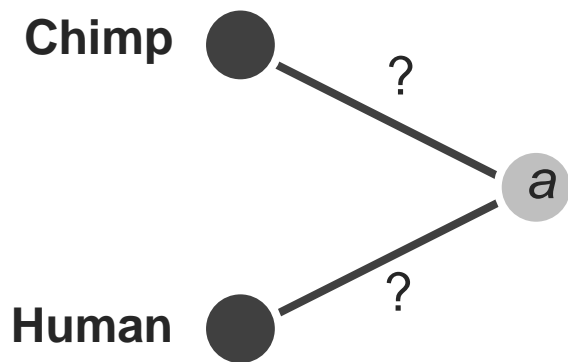
	Chimp	Human	Seal	Whale	<i>m</i>
Chimp	0	3	6	4	4
Human	3	0	7	5	5
Seal	6	7	0	2	2
Whale	4	5	2	0	0
<i>m</i>	4	5	2	0	0



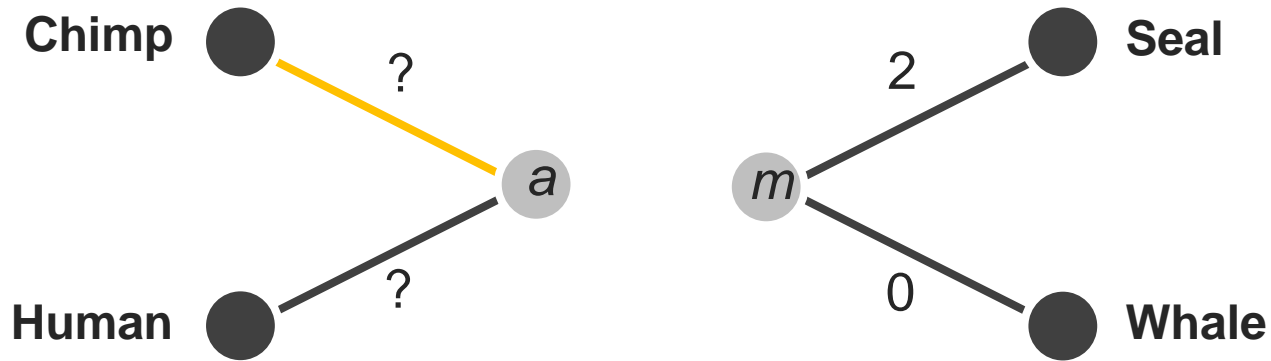
	Chimp	Human	<i>m</i>
Chimp	0	3	4
Human	3	0	5
<i>m</i>	4	5	0



	Chimp	Human	<i>m</i>
Chimp	0	3	4
Human	3	0	5
<i>m</i>	4	5	0

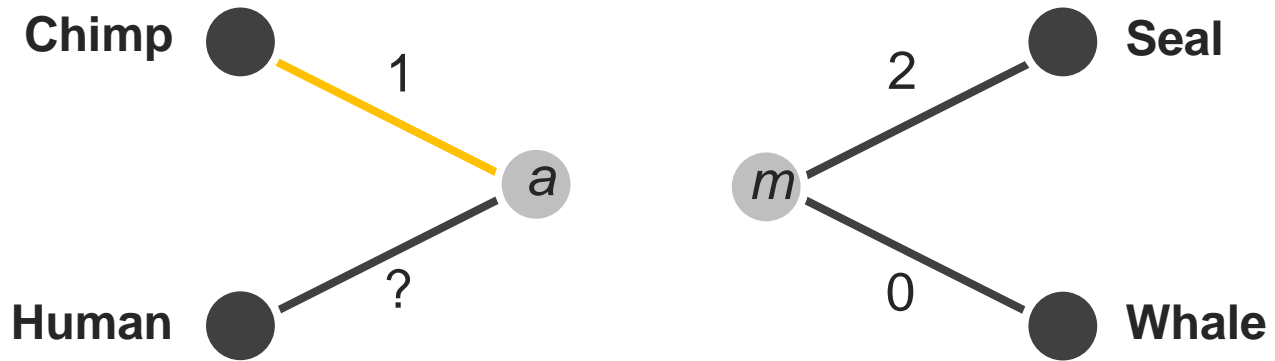


	Chimp	Human	<i>m</i>
Chimp	0	3	4
Human	3	0	5
<i>m</i>	4	5	0



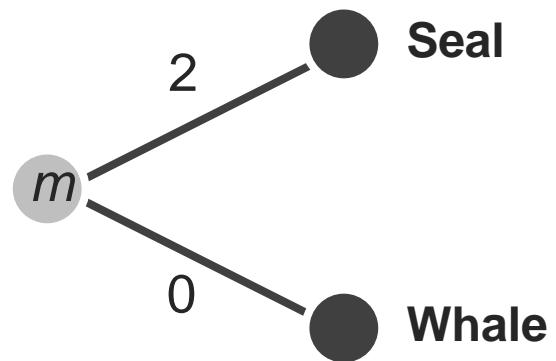
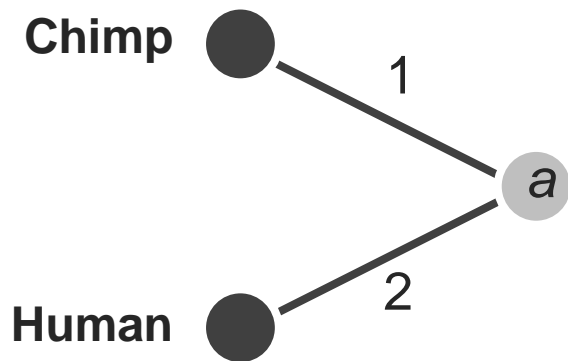
$$d_{\text{Chimp},a} = (D_{\text{Chimp},m} + D_{\text{Chimp},\text{Human}} - D_{\text{Human},m}) / 2$$

	Chimp	Human	m
Chimp	0	3	4
Human	3	0	5
m	4	5	0

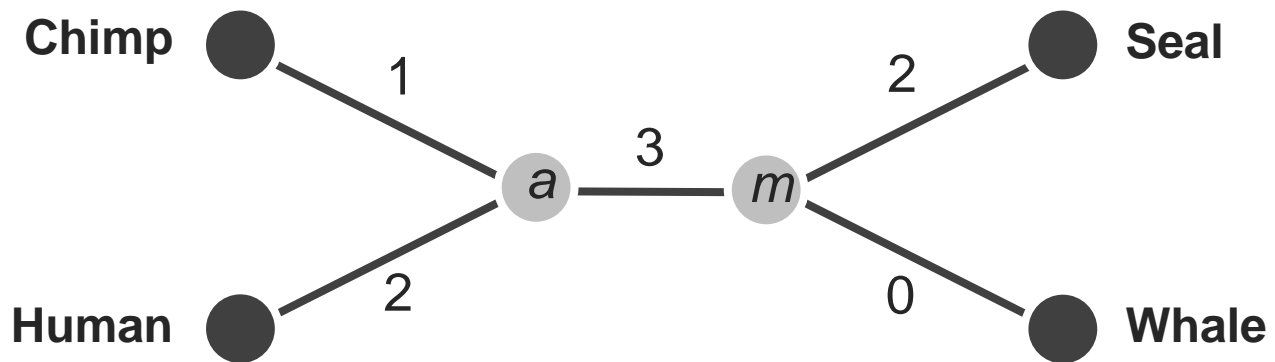


$$d_{\text{Chimp},a} = 1$$

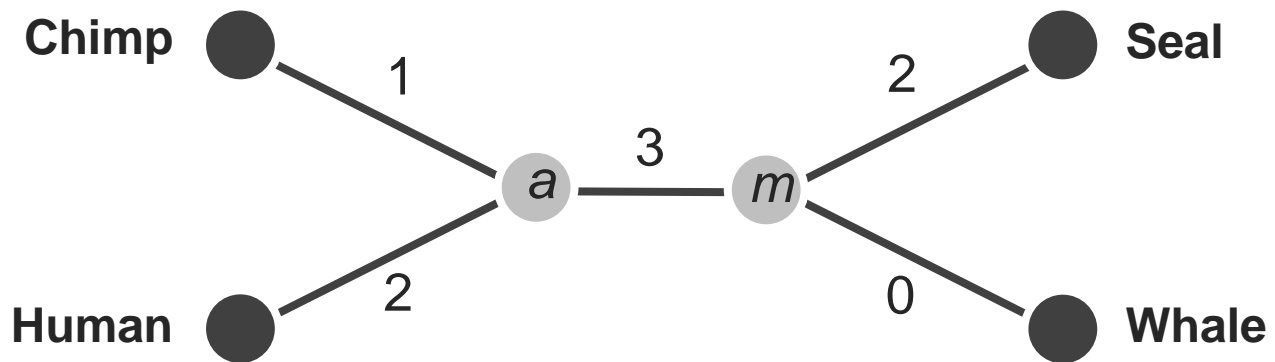
	Chimp	Human	<i>m</i>
Chimp	0	3	4
Human	3	0	5
<i>m</i>	4	5	0



	Chimp	Human	<i>m</i>
Chimp	0	3	4
Human	3	0	5
<i>m</i>	4	5	0



	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



Primenimo ovaj pristup na sledeću matricu:

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0

Pregled

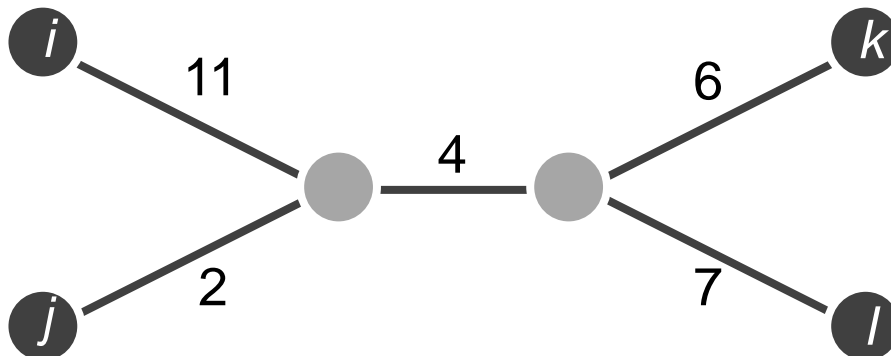
- Izbijanje epidemije
- Transformacija matrice rastojanja u evolutivno stablo
- Prema algoritmu za rekonstrukciju filogenetskog stabla na osnovu rastojanja
- **AdditivePhylogeny algoritam**
- Metod najmanjih kvadrata
- Ultrametrična evolutivna stabla
- Neighbour-Joining algoritam
- Rekonstrukcija stabla na osnovu karakteristika
- Problem male parsimonije
- Problem velike parsimonije

Zašto naš algoritam ne radi za ovu matricu?

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0

Zašto naš algoritam ne radi za ovu matricu?

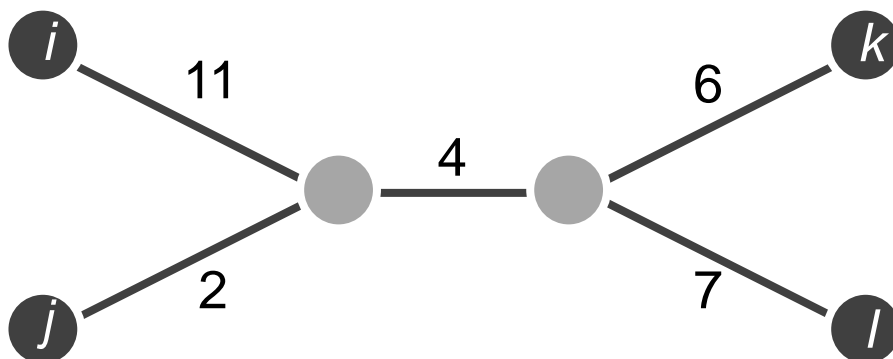
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0



Zašto naš algoritam ne radi za ovu matricu?

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0

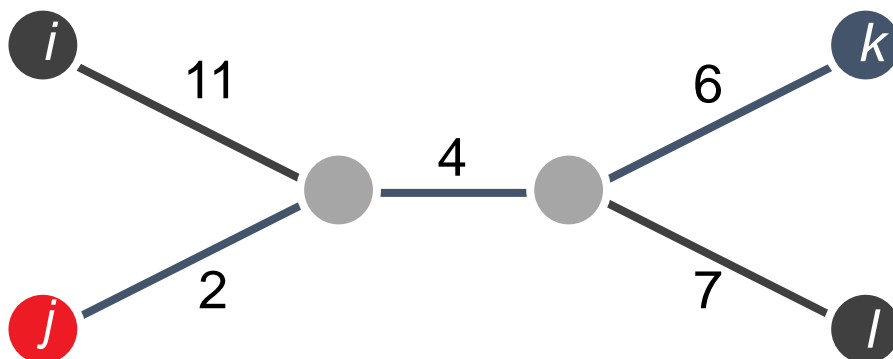
minimalni
element je
 $D_{j,k}$



Zašto naš algoritam ne radi za ovu matricu?

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0

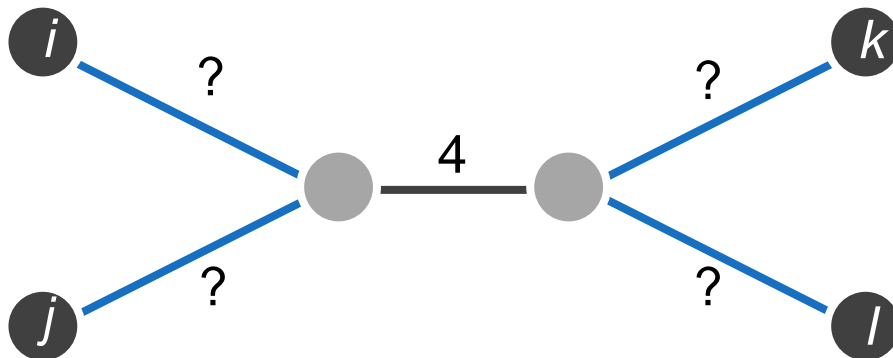
minimalni
element je
 $D_{j,k}$



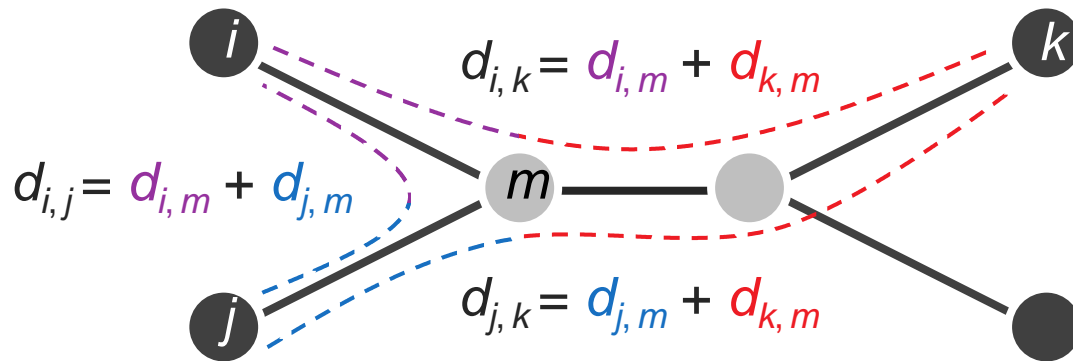
j i *k* nisu
susedi!

Od suseda do spoljnih grana

Umesto traženja **suseda**, pokušajmo da izračunamo dužinu **spoljnih grana**, onih grana koje vode do listova.



Od suseda do spoljnih grana



$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$$d_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

$$\therefore d_{i,m} = D_{i,k} - (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

$$d_{i,m} = (D_{i,k} + D_{i,j} - D_{j,k}) / 2$$

Važi pod pretpostavkom
da su i i j susedni
listovi

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:

LimbLength(*i*) je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima *j* i *k*.

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:
 $LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:
 $LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

$$(D_{chimp, human} + D_{chimp, seal} - D_{human, seal}) / 2 = (3 + 6 - 7) / 2 = 1$$

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:
 $LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

$$(D_{chimp, human} + D_{chimp, seal} - D_{human, seal}) / 2 = (3 + 6 - 7) / 2 = 1$$

$$(D_{chimp, human} + D_{chimp, whale} - D_{human, whale}) / 2 = (3 + 4 - 5) / 2 = 1$$

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:

$LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

$$(D_{chimp, human} + D_{chimp, seal} - D_{human, seal}) / 2 = (3 + 6 - 7) / 2 = 1$$

$$(D_{chimp, human} + D_{chimp, whale} - D_{human, whale}) / 2 = (3 + 4 - 5) / 2 = 1$$

$$(D_{chimp, whale} + D_{chimp, seal} - D_{whale, seal}) / 2 = (6 + 4 - 2) / 2 = 4$$

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:

$LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0

$$(D_{\text{human, chimp}} + D_{\text{chimp, seal}} - D_{\text{human, seal}}) / 2 = (3 + 6 - 7) / 2 = \mathbf{1}$$

$$(D_{\text{human, chimp}} + D_{\text{chimp, whale}} - D_{\text{human, whale}}) / 2 = (3 + 4 - 5) / 2 = \mathbf{1}$$

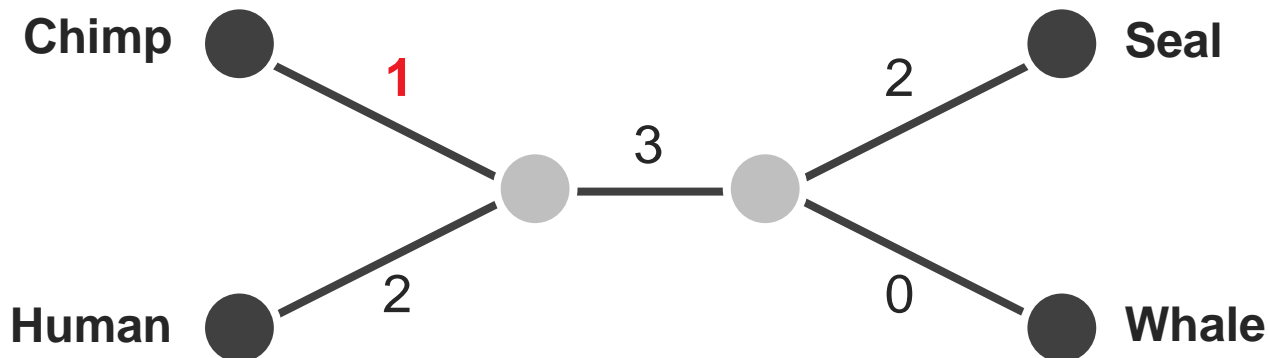
$$(D_{\text{whale, chimp}} + D_{\text{chimp, seal}} - D_{\text{whale, seal}}) / 2 = (6 + 4 - 2) / 2 = 4$$

Računanje dužine spoljnih grana

Teorema o dužini spoljnih grana:

$LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

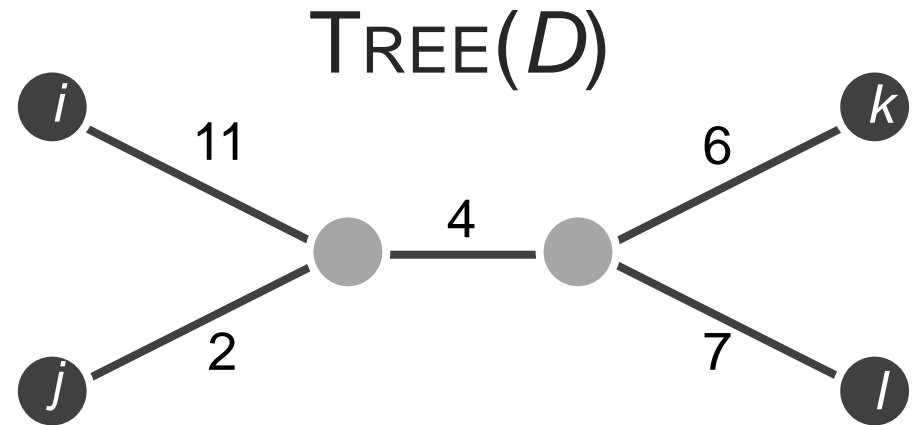
	Chimp	Human	Seal	Whale
Chimp	0	3	6	4
Human	3	0	7	5
Seal	6	7	0	2
Whale	4	5	2	0



Additive Phylogeny

D

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0



Additive Phylogeny

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	
<i>D</i>	<i>i</i>	0	13	21	22
	<i>j</i>	13	0	12	13
	<i>k</i>	21	12	0	13
	<i>l</i>	22	13	13	0

1. Izaberemo proizvoljno list, npr. *j*.

AdditivePhyLogeny

D

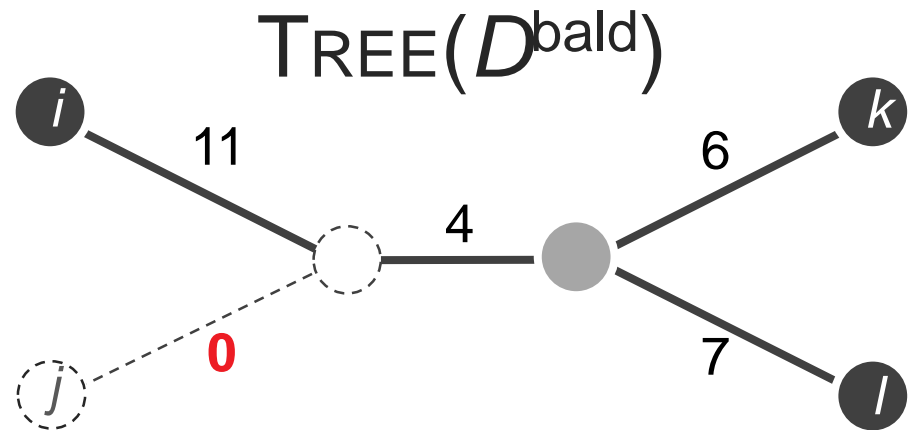
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	13	21	22
<i>j</i>	13	0	12	13
<i>k</i>	21	12	0	13
<i>l</i>	22	13	13	0

$$\text{LimbLength}(j) = 2$$

2. Izračunamo dužinu njegove krajnje grane, $\text{LimbLength}(j)$.

Additive Phylogeny

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0



3. Oduzmemo $LimbLength(j)$ od svake grane i dobijemo matricu D^{bald} u kojoj do lista *j* vodi ogoljena (**bold**) grana (dužine 0).

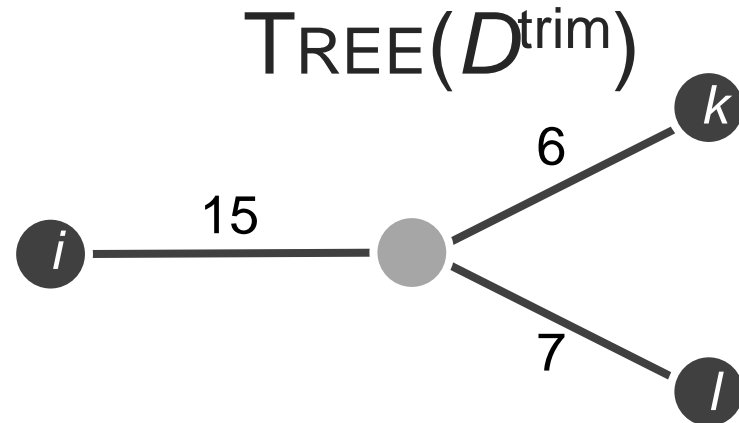
Additive Phylogeny

		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
D^{trim}	<i>i</i>	0	11	21	22
	<i>j</i>	11	0	10	11
	<i>k</i>	21	10	0	13
	<i>l</i>	22	11	13	0

4. Uklonimo j -ti red i kolonu iz matrice i dobijemo $(n - 1) \times (n - 1)$ matricu D^{trim} .

Additive Phylogeny

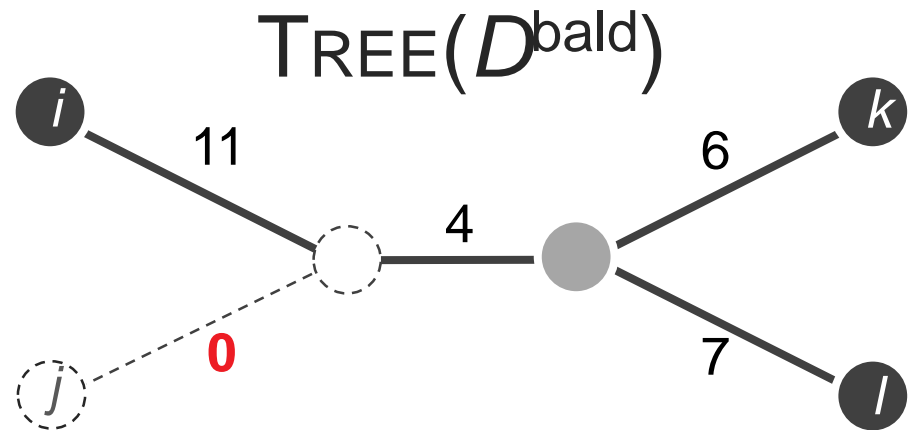
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0



5. Konstruišemo $Tree(D^{\text{trim}})$.

Additive Phylogeny

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>D</i> ^{bald} <i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0

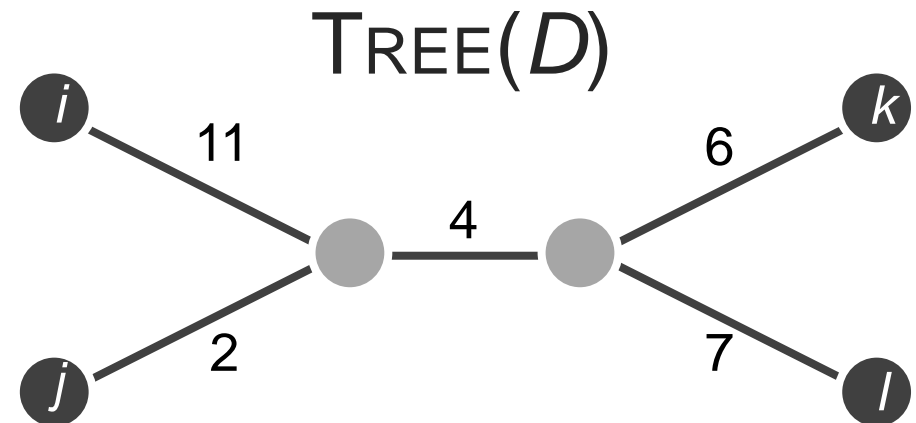


6. Identifikujemo tačku u $Tree(D^{\text{trim}})$ gde list *j* treba da se nalazi.

Additive Phylogeny

D

	i	j	k	l
i	0	13	21	22
j	13	0	12	13
k	21	12	0	13
l	22	13	13	0



$$\text{LimbLength}(j) = 2$$

7. Dodamo list j povezujući ga granom dužine $\text{LimbLength}(j)$ kako bismo formirali $\text{Tree}(D)$.

AdditivePhyLogeny

AdditivePhyLogeny(D):

1. Izaberemo proizvoljno list, npr. j .
2. Izračunamo dužinu njegove krajnje grane, $\text{LimbLength}(j)$.
3. Oduzmemo $\text{LimbLength}(j)$ od svake grane i dobijemo matricu D_{bold} u kojoj do lista j vodi ogoljena (bold) grana (dužine 0).
4. Uklonimo j -ti red i kolonu iz matrice i dobijemo $(n - 1) \times (n - 1)$ matricu D_{trim} .
5. Konstruišemo $\text{Tree}(D_{\text{trim}})$.
6. Identifikujemo tačku u $\text{Tree}(D_{\text{trim}})$ gde list j treba da se nalazi.
7. Dodamo list j povezujući ga granom dužine $\text{LimbLength}(j)$ kako bismo formirali $\text{Tree}(D)$.

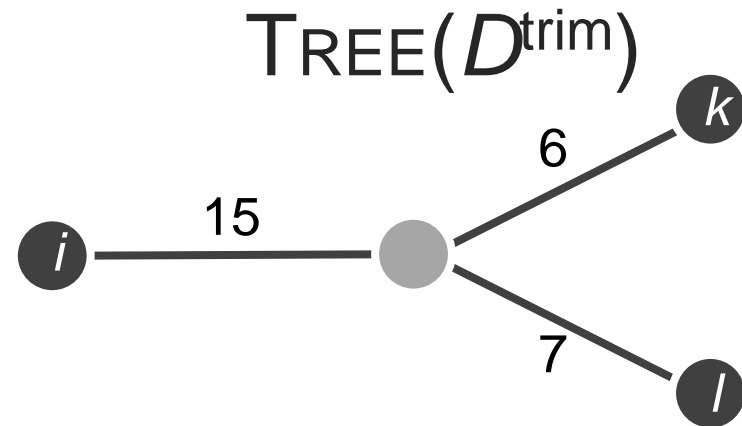
Additive Phylogeny

Additive Phylogeny(D):

1. Izaberemo proizvoljno list, npr. j .
2. Izračunamo dužinu njegove krajnje grane, $\text{LimbLength}(j)$.
3. Oduzmemo $\text{LimbLength}(j)$ od svake grane i dobijemo matricu D_{bold} u kojoj do lista j vodi ogoljena (bold) grana (dužine 0).
4. Uklonimo j -ti red i kolonu iz matrice i dobijemo $(n - 1) \times (n - 1)$ matricu D_{trim} .
5. Konstruišemo $\text{Tree}(D_{\text{trim}})$.
- 6. Identifikujemo tačku u $\text{Tree}(D_{\text{trim}})$ gde list j treba da se nalazi.**
7. Dodamo list j povezujući ga granom dužine $\text{LimbLength}(j)$ kako bismo formirali $\text{Tree}(D)$.

Povezivanje lista

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0

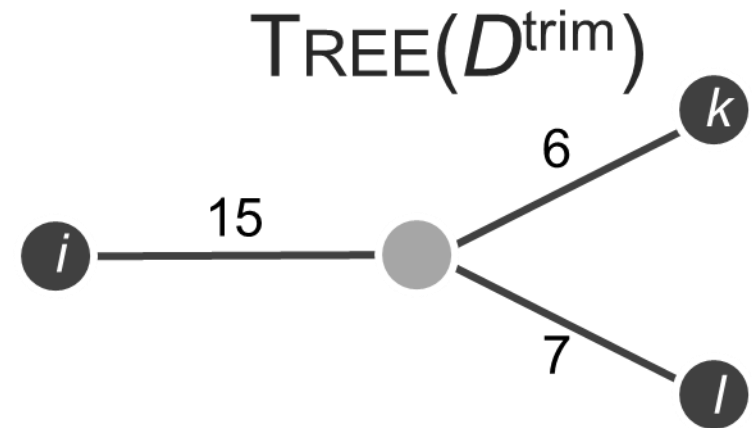


Teorema o dužini spoljnih grana:

$LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

Povezivanje lista

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0



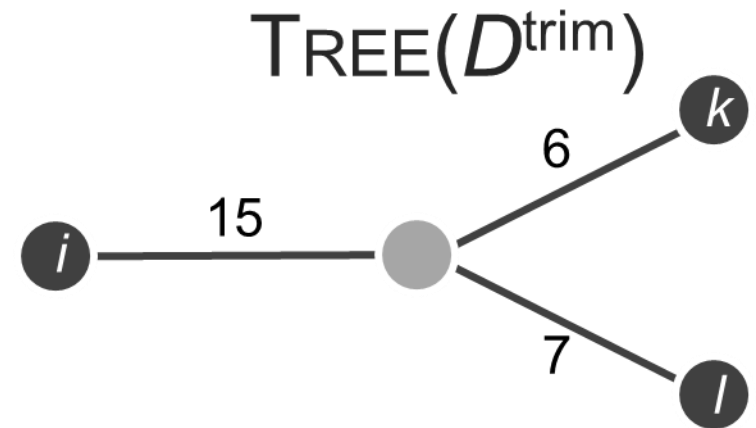
Teorema o dužini spoljnih grana:

$LimbLength(i)$ je jednako minimalnoj vrednosti $(D_{i,k} + D_{i,j} - D_{j,k})/2$ po svim listovima j i k .

$$(D_{i,j}^{\text{bald}} + D_{j,k}^{\text{bald}} - D_{i,k}^{\text{bald}})/2 = 0$$

Povezivanje lista

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0

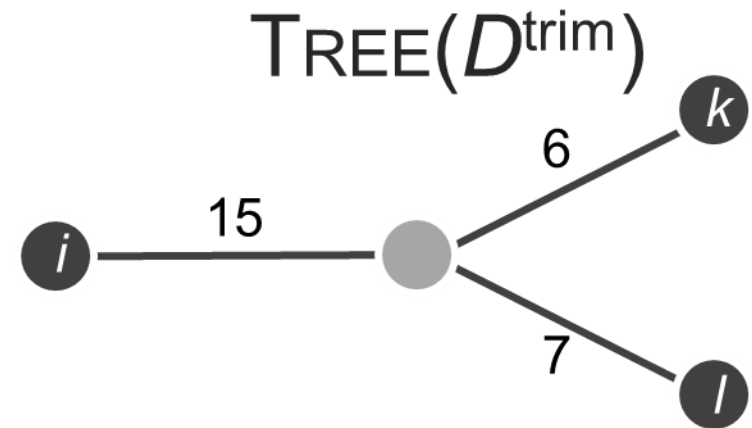


$$(D^{\text{bald}}_{i,j} + D^{\text{bald}}_{j,k} - D^{\text{bald}}_{i,k})/2 = 0$$

$$D^{\text{bald}}_{i,j} + D^{\text{bald}}_{j,k} = D^{\text{bald}}_{i,k}$$

Povezivanje lista

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	11	21	22
<i>j</i>	11	0	10	11
<i>k</i>	21	10	0	13
<i>l</i>	22	11	13	0



Tačka povezivanja za list j je na putanji između listova i i k na rastojanju $D^{\text{bald}}_{i,j}$ od i .

$$D^{\text{bald}}_{i,j} + D^{\text{bald}}_{j,k} = D^{\text{bald}}_{i,k}$$

Matrica rastojanja za *spike* protein

	Cow	Pig	Horse	Mouse	Dog	Cat	Turkey	Civet	Human
Cow	0	226	249	436	958	916	730	787	785
Pig	226	0	292	436	903	905	744	802	813
Horse	249	292	0	426	927	907	735	795	791
Mouse	436	436	426	0	917	946	725	767	782
Dog	958	903	927	917	0	706	730	844	846
Cat	916	905	907	946	706	0	736	840	836
Turkey	730	744	735	725	730	736	0	763	760
Civet	787	802	795	767	844	840	763	0	16
Human	785	813	791	782	846	836	760	16	0

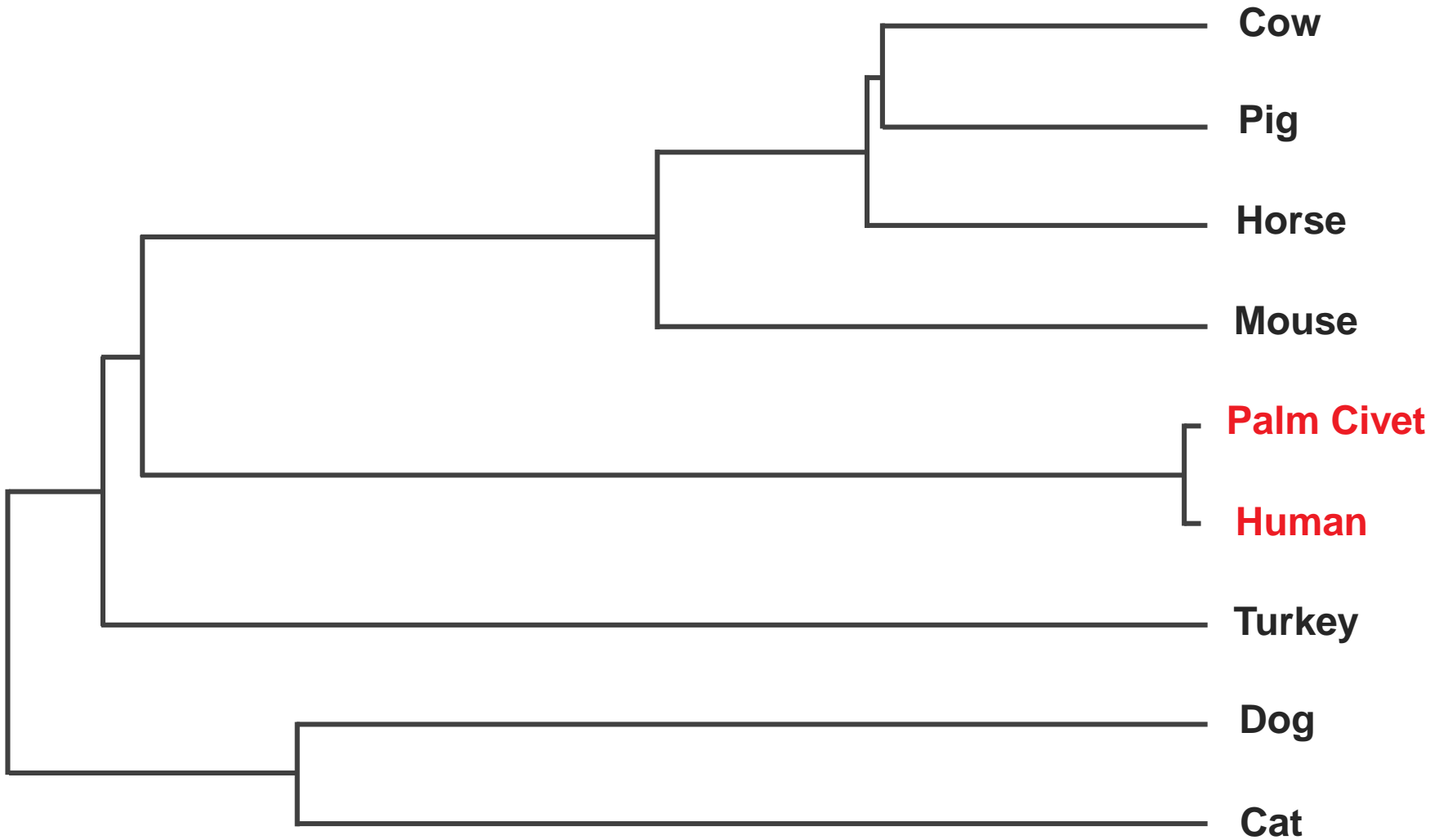
Nije aditivna! Malo je izmenimo...

Matrica rastojanja za *spike* protein

	Cow	Pig	Horse	Mouse	Dog	Cat	Turkey	Civet	Human
Cow	0	295	306	497	1081	1091	1003	956	954
Pig	295	0	309	500	1084	1094	1006	959	957
Horse	306	309	0	489	1073	1083	995	948	946
Mouse	497	500	489	0	1092	1102	1014	967	965
Dog	1081	1084	1073	1092	0	818	1056	1053	1051
Cat	1091	1094	1083	1102	818	0	1066	1063	1061
Turkey	1003	1006	995	1014	1056	1066	0	975	973
Civet	956	959	948	967	1053	1063	975	0	16
Human	954	957	946	965	1051	1061	973	16	0

Koja životinja nam je donela SARS?

Filogeneza coronavirus-a











Matrica rastojanja za *spike* protein

	Cow	Pig	Horse	Mouse	Dog	Cat	Turkey	Civet	Human
Cow	0	226	249	436	958	916	730	787	785
Pig	226	0	292	436	903	905	744	802	813
Horse	249	292	0	426	927	907	735	795	791
Mouse	436	436	426	0	917	946	725	767	782
Dog	958	903	927	917	0	706	730	844	846
Cat	916	905	907	946	706	0	736	840	836
Turkey	730	744	735	725	730	736	0	763	760
Civet	787	802	795	767	844	840	763	0	16
Human	785	813	791	782	846	836	760	16	0

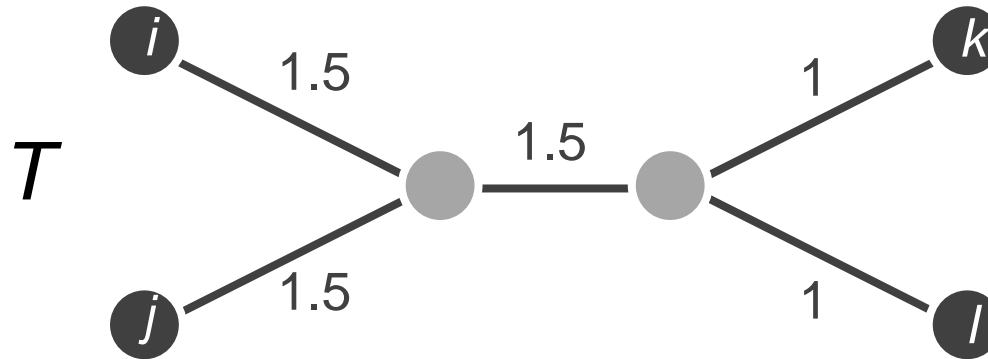
Šta ako matrica nije aditivna?

Pregled

- Izbijanje epidemije
- Transformacija matrice rastojanja u evolutivno stablo
- Prema algoritmu za rekonstrukciju filogenetskog stabla na osnovu rastojanja
- AdditivePhylogeny algoritam
- **Metod najmanjih kvadrata**
- Ultrametrična evolutivna stabla
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- Problem velike parsimonije

Metod najmanjih kvadrata

- Šta ako imamo matricu koja nije aditivna?

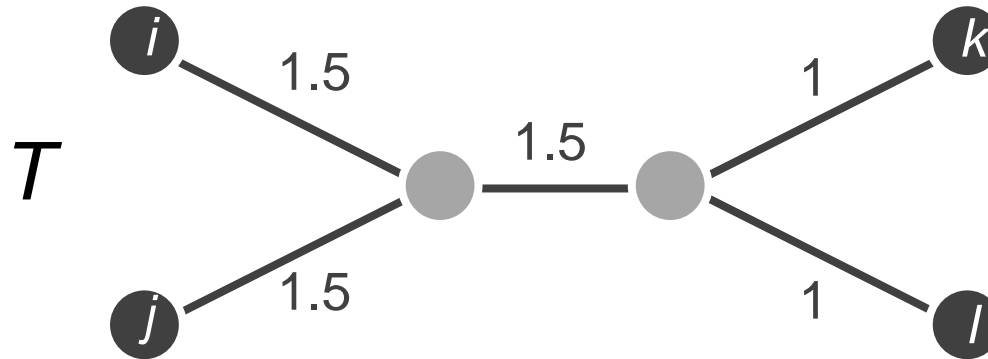


D

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

Metod najmanjih kvadrata

- Šta ako imamo matricu koja nije aditivna?
- Uzmemo u obzir njenu aproksimaciju nekom aditivnom matricom



D

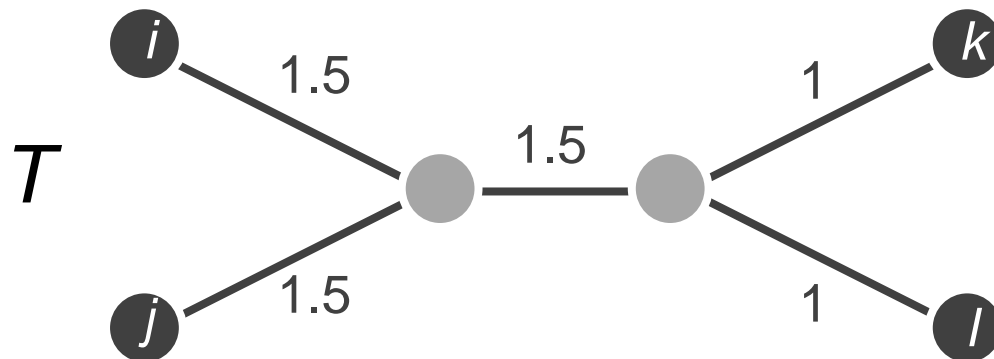
	i	j	k	l
i	0	3	4	3
j	3	0	4	5
k	4	4	0	2
l	3	5	2	0

d

	i	j	k	l
i	0	3	4	4
j	3	0	4	4
k	4	4	0	2
l	4	4	2	0

Metod najmanjih kvadrata

- Šta ako imamo matricu koja nije aditivna?
- Uzmemo u obzir njenu aproksimaciju nekom aditivnom matricom
- Koja je najbolja aproksimacija?



D

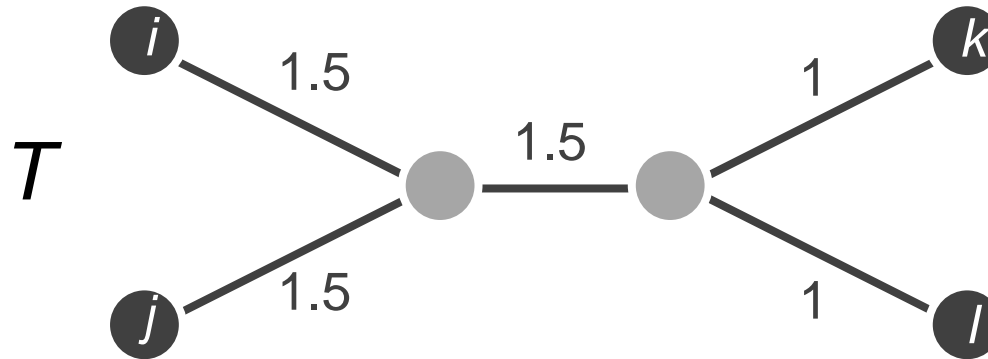
	i	j	k	l
i	0	3	4	3
j	3	0	4	5
k	4	4	0	2
l	3	5	2	0

d

	i	j	k	l
i	0	3	4	4
j	3	0	4	4
k	4	4	0	2
l	4	4	2	0

Metod najmanjih kvadrata

$$Discrepancy(T, D) = \sum_{1 \leq i < j \leq n} (d_{i,j}(T) - D_{i,j})^2$$



D

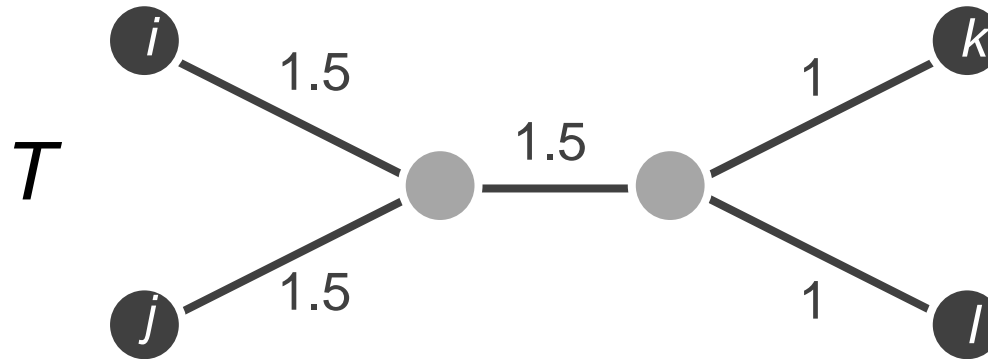
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

d

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	4
<i>j</i>	3	0	4	4
<i>k</i>	4	4	0	2
<i>l</i>	4	4	2	0

Metod najmanjih kvadrata

$$Discrepancy(T, D) = \sum_{1 \leq i < j \leq n} (d_{i,j}(T) - D_{i,j})^2 = 1^2 + 1^2 = 2$$



D

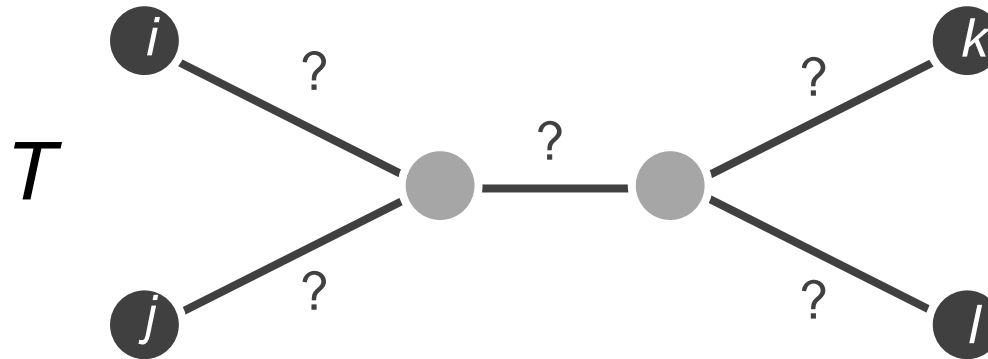
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

d

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	4
<i>j</i>	3	0	4	4
<i>k</i>	4	4	0	2
<i>l</i>	4	4	2	0

Metod najmanjih kvadrata

Šta ako aditivna matrica nije poznata: Dodeliti dužine granama u stablu T tako da veličina $Discrepancy(T, D)$ bude minimalna.



D

	i	j	k	l
i	0	3	4	3
j	3	0	4	5
k	4	4	0	2
l	3	5	2	0

d

	i	j	k	l
i	0	?	?	?
j	?	0	?	?
k	?	?	0	?
l	?	?	?	0

Metod najmanjih kvadrata

- U opštem slučaju, za stablo date topologije postoji algoritam polinomijalne složenosti koji će dodeliti dužine granama stabla tako da diskrepanca bude minimalna
- Međutim, u praktičnim primenama neće biti poznata topologija stabla pa stoga moramo računati minimum po svim mogućim stablima
- Sa dodavanjem svakog lista u stablo, broj različitih topologija stabala raste eksponencijalno
- Problem minimizacije diskrepance po svim mogućim stablima je NP kompletan
- U nastavku, razmotrićemo dve heuristike za konstrukciju stabla na osnovu neaditivnih matrica

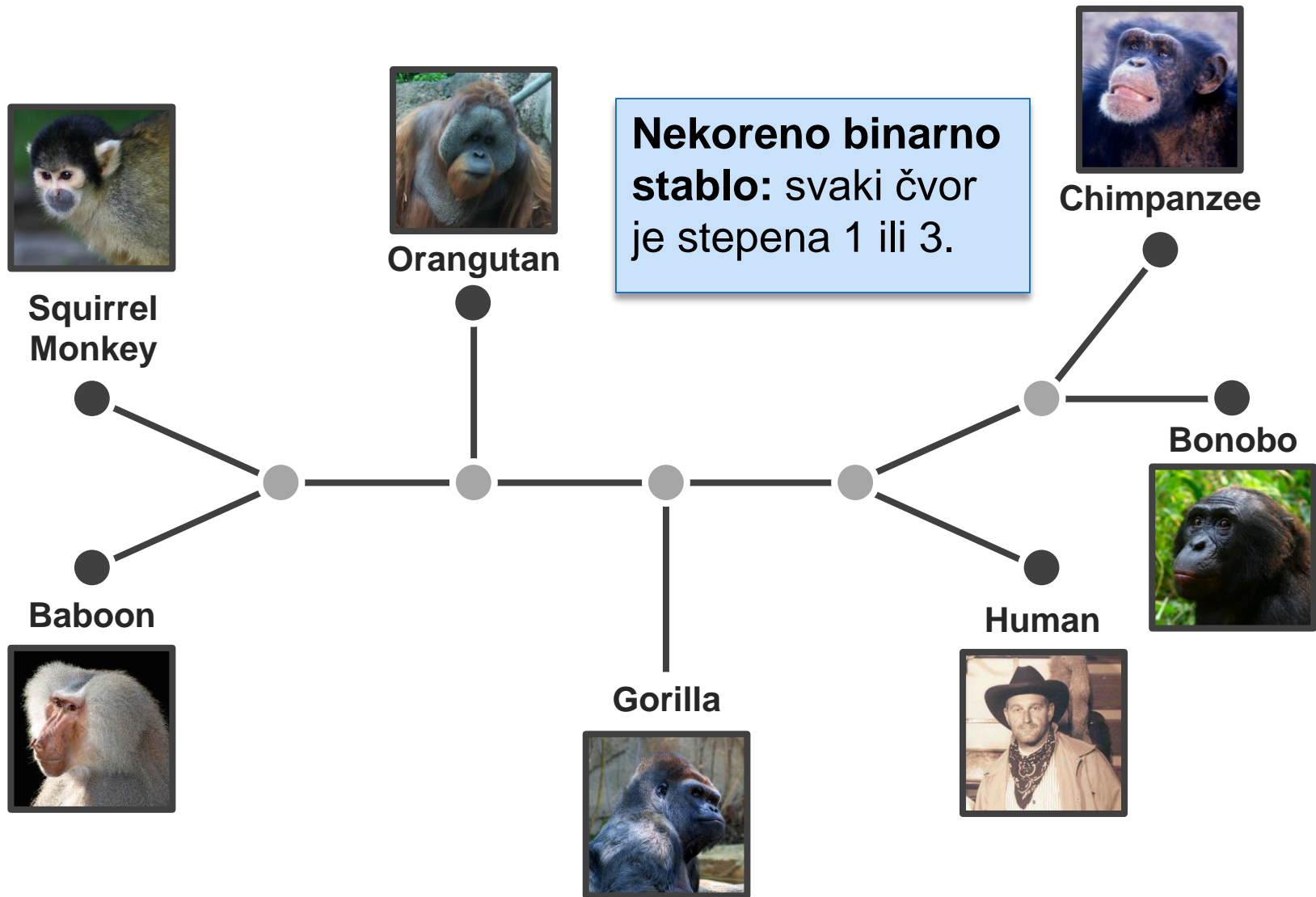
Pregled

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Modelovanje specijacije

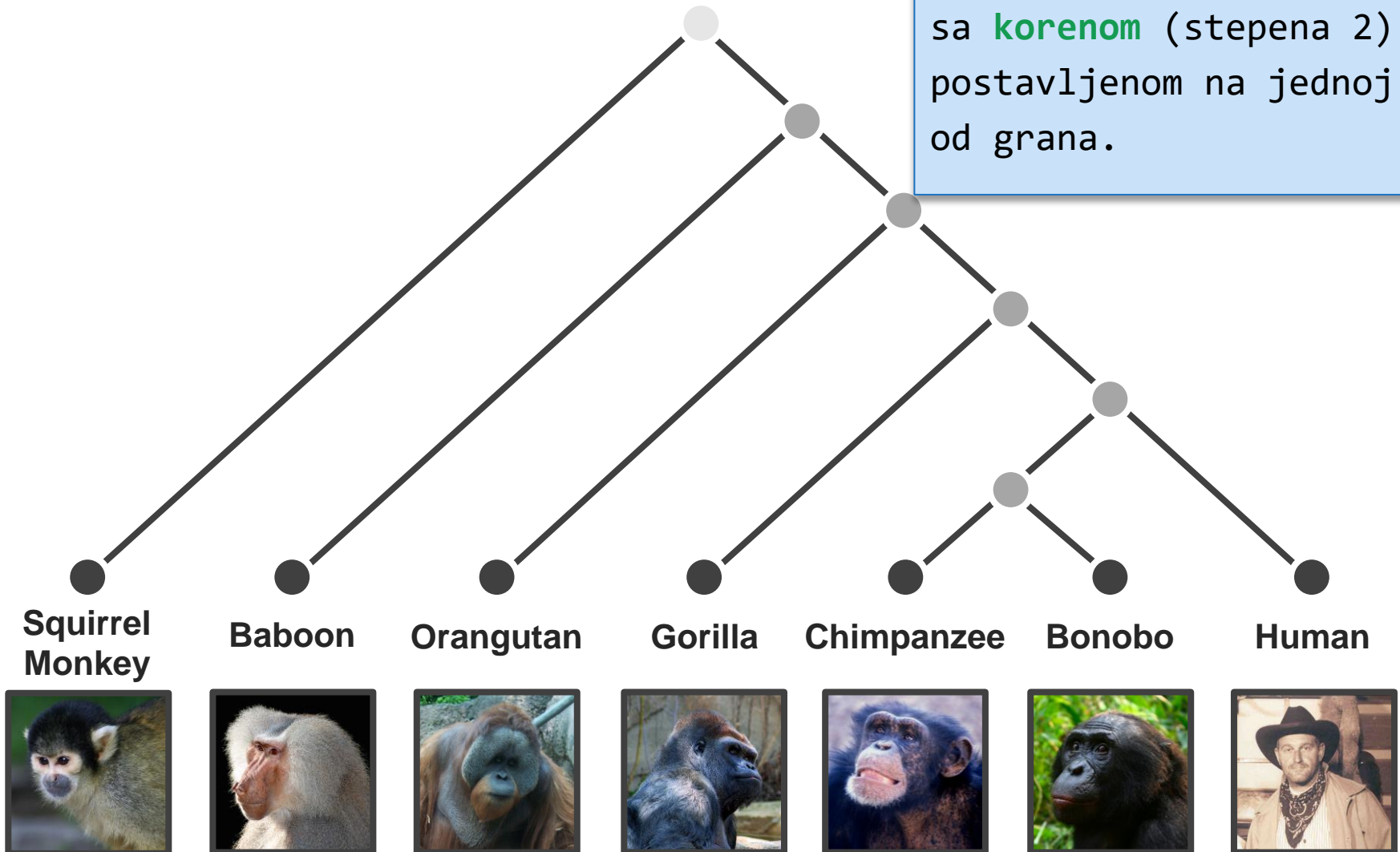
U praktičnim primenama, istraživači često pretpostavljaju da svaki unutrašnji čvor odgovara *specijaciji* kada se jedna vrsta deli na dve.

Modelovanje specijacije



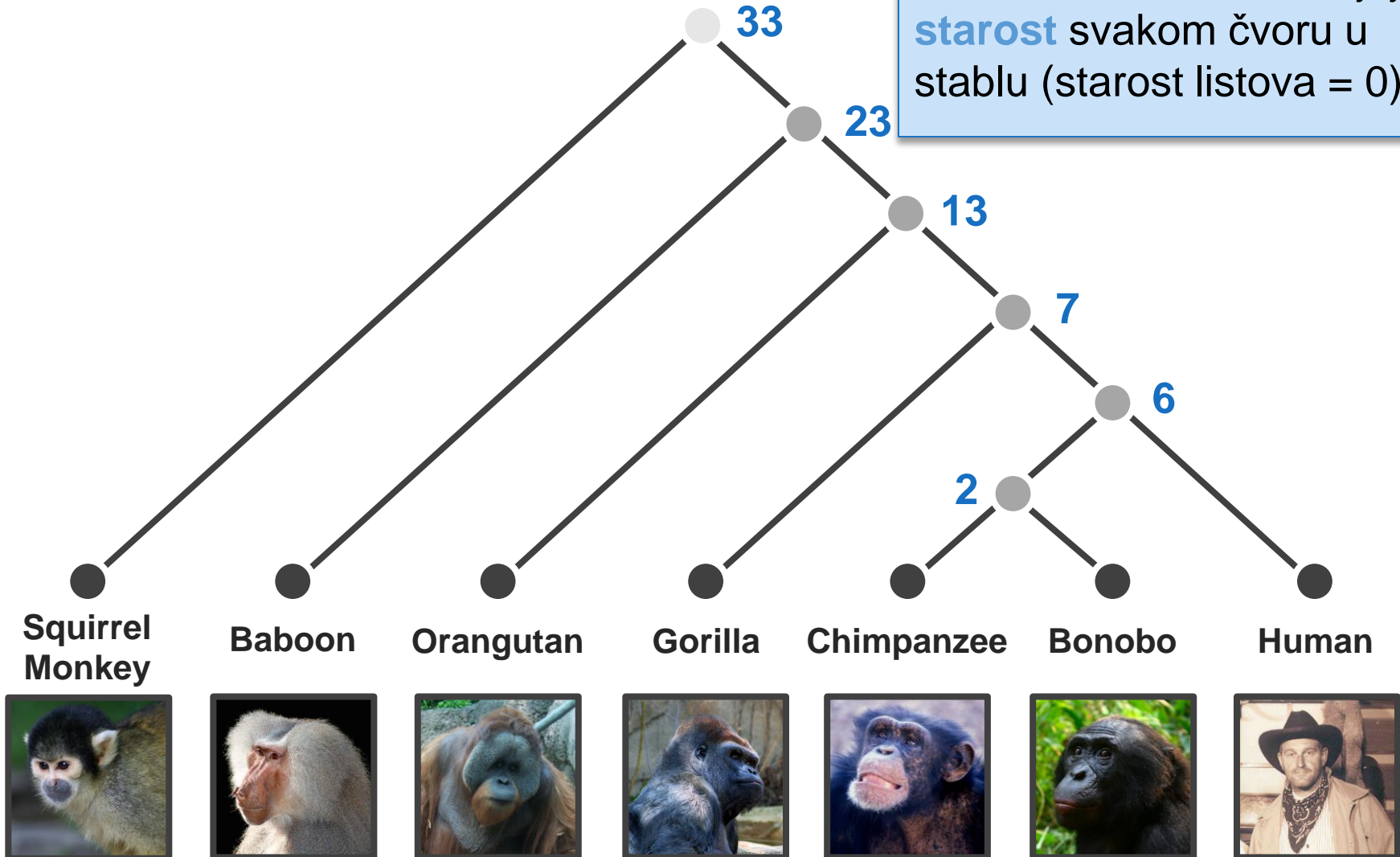
Modelovanje specijacije

Koreno binarno stablo:
nekoreno binarno stablo
sa **korenom** (stepena 2)
postavljenom na jednoj
od grana.



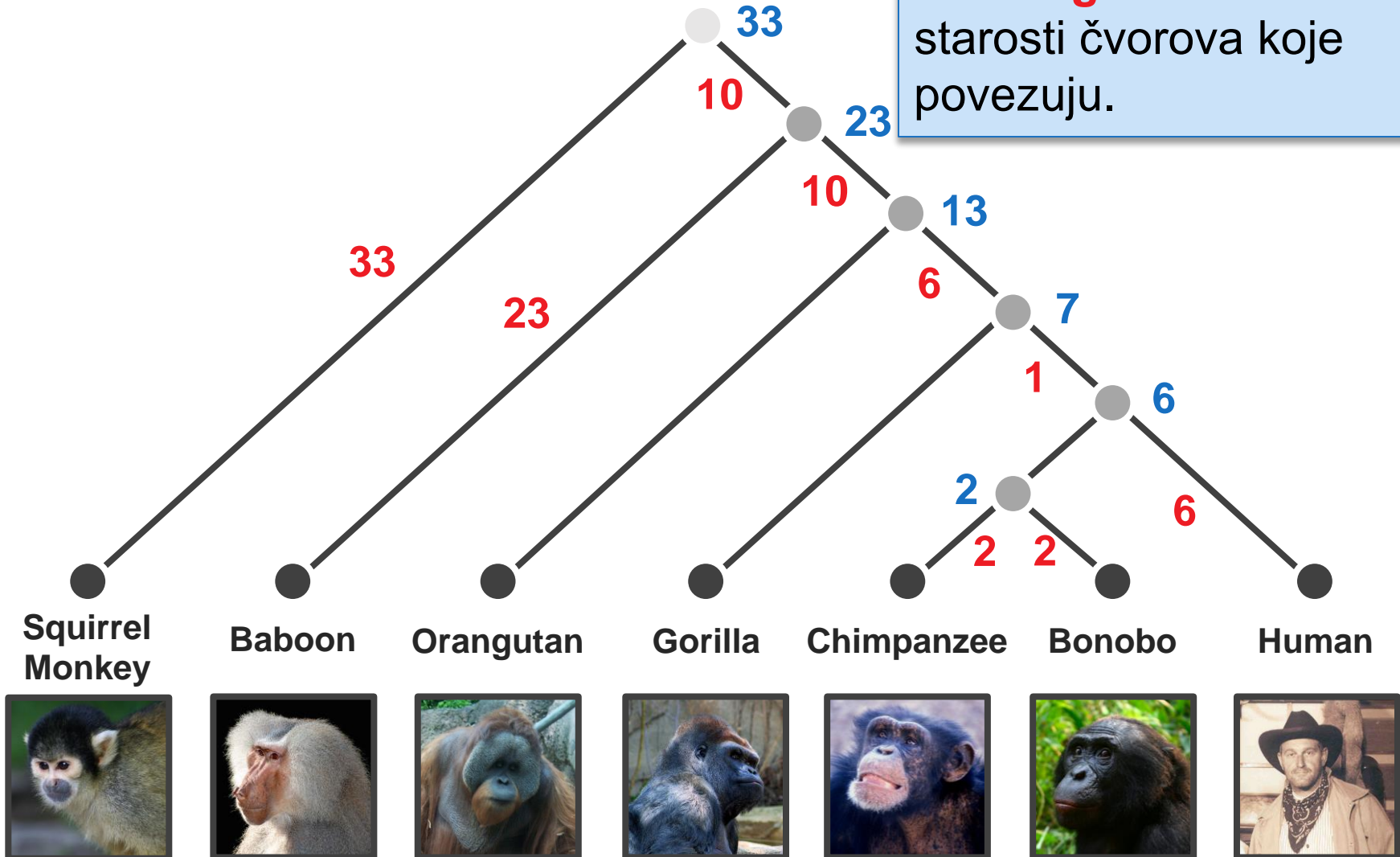
Ultrametrična stabla

Molekularni sat: dodeljuje **starost** svakom čvoru u stablu (starost listova = 0).



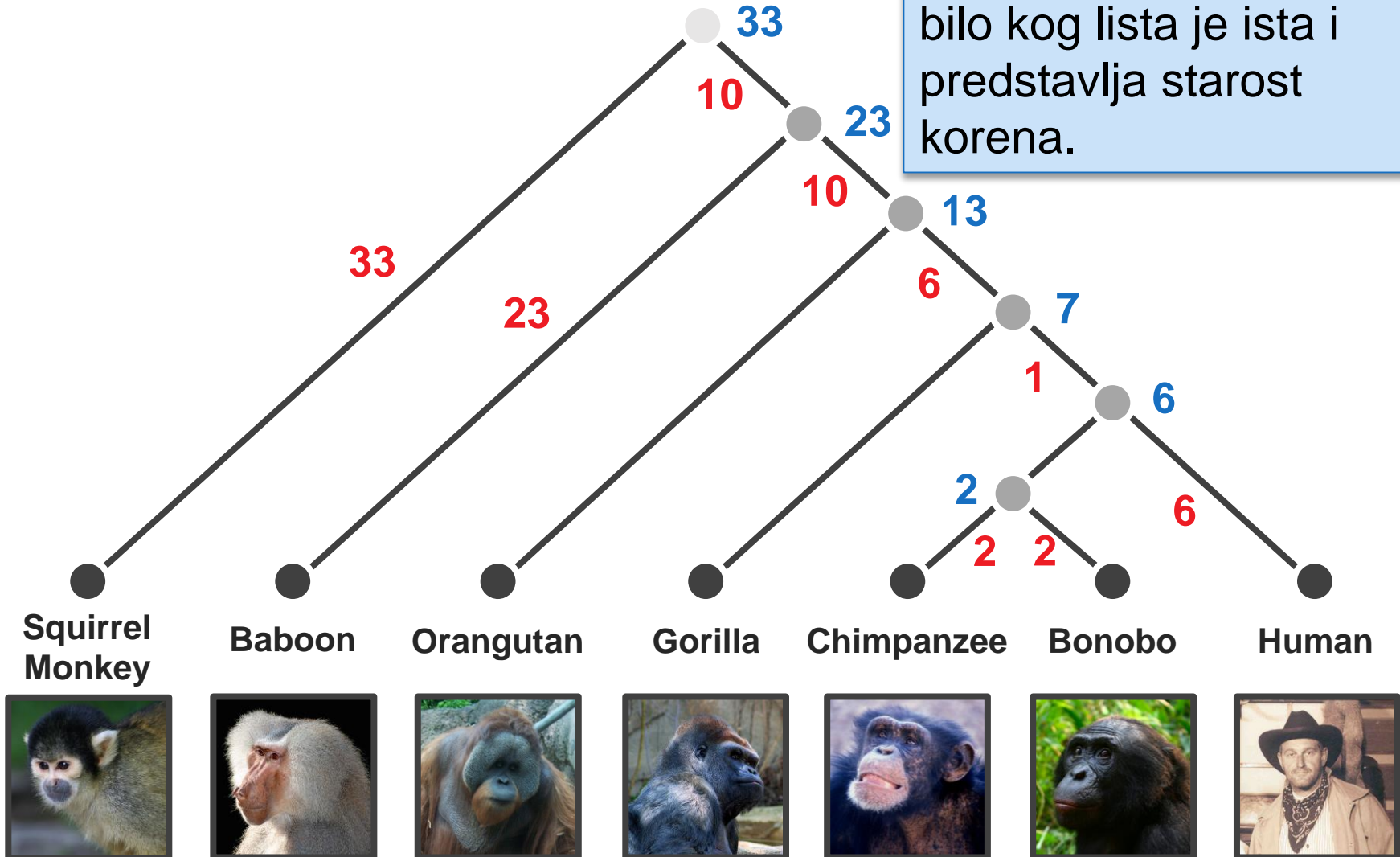
Ultrametrična stabla

Težine grana: razlika u starosti čvorova koje povezuju.



Ultrametrična stabla

Ultrametrično stablo:
udaljenost od korena do bilo kog lista je ista i predstavlja starost korena.



UPGMA: heurističko klasterovanje

1. Formirati klaster za svaku današnju vrstu. Svaki klaster sadrži jedan list

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0



UPGMA: heurističko klasterovanje

2. Naći dva najbliža klastera C_1 i C_2 na osnovu prosečnog rastojanja između njihovih članova

$$D_{\text{avg}}(C_1, C_2) = \sum_{i \text{ in } C_1, j \text{ in } C_2} D_{i,j} / |C_1| \cdot |C_2|$$

gde $|C|$ označava broj elemenata u klasteru C .

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0



UPGMA: heurističko klasterovanje

3. Spojiti C_1 i C_2 u jedinstveni klaster C .

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0

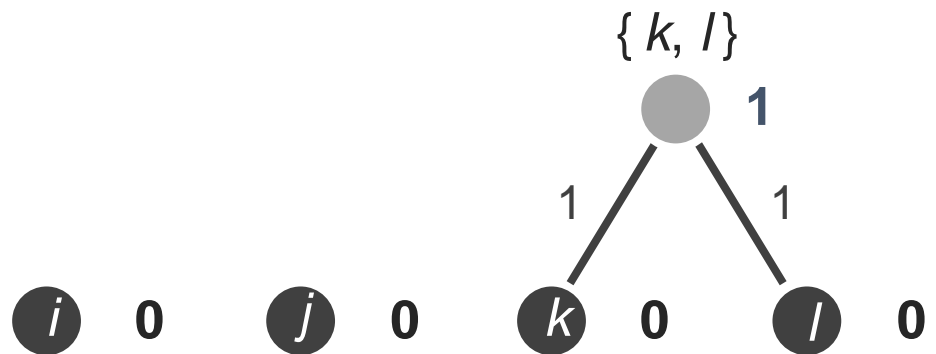


{*k*, *l*}

UPGMA: heurističko klasterovanje

4. Formirati novi čvor za klaster i granama povezati ga za čvorovima.
Postaviti starost čvora C na $D_{\text{avg}}(C_1, C_2)/2$.

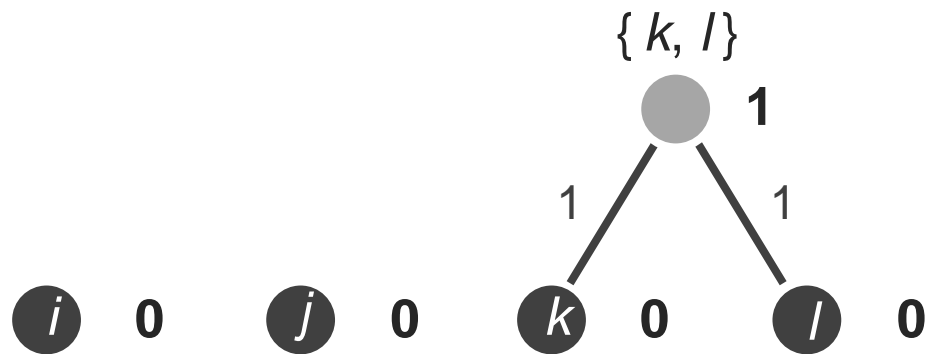
	i	j	k	l
i	0	3	4	3
j	3	0	4	5
k	4	4	0	2
l	3	5	2	0



UPGMA: heurističko klasterovanje

5. Ažurirati matricu rastojanja tako što ubacimo novi čvor, izbacimo čvorove koje on sadrži i izračunamo rastojanja kao prosečna rastojanja između svaka dva para klastera.

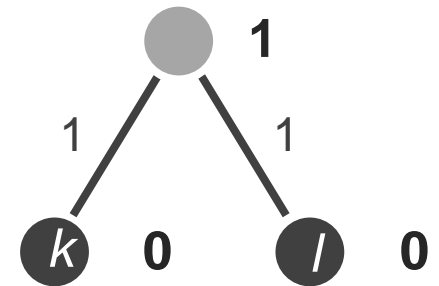
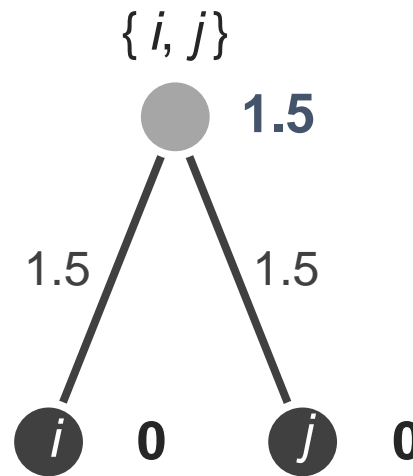
	i	j	$\{k, l\}$
i	0	3	3.5
j	3	0	4.5
$\{k, l\}$	3.5	4.5	0



UPGMA: heurističko klasterovanje

6. Iteriramo dok ne dođemo do jednog klastera koji sadrži sve vrste.

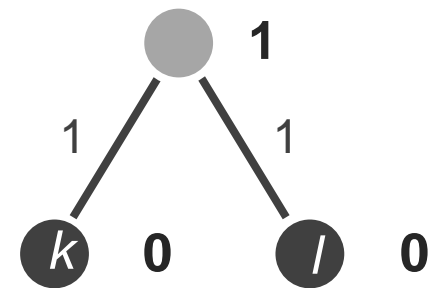
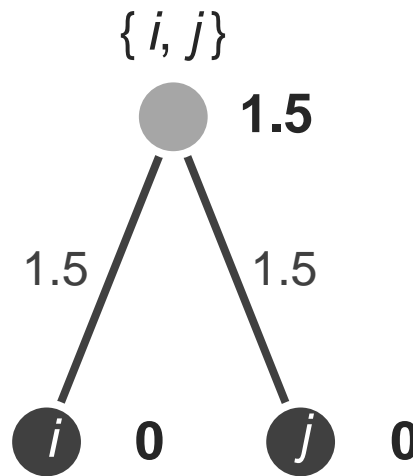
	<i>i</i>	<i>j</i>	{ <i>k</i> , <i>l</i> }
<i>i</i>	0	3	3.5
<i>j</i>	3	0	4.5
{ <i>k</i> , <i>l</i> }	3.5	4.5	0



UPGMA: heurističko klasterovanje

6. Iteriramo dok ne dođemo do jednog klastera koji sadrži sve vrste.

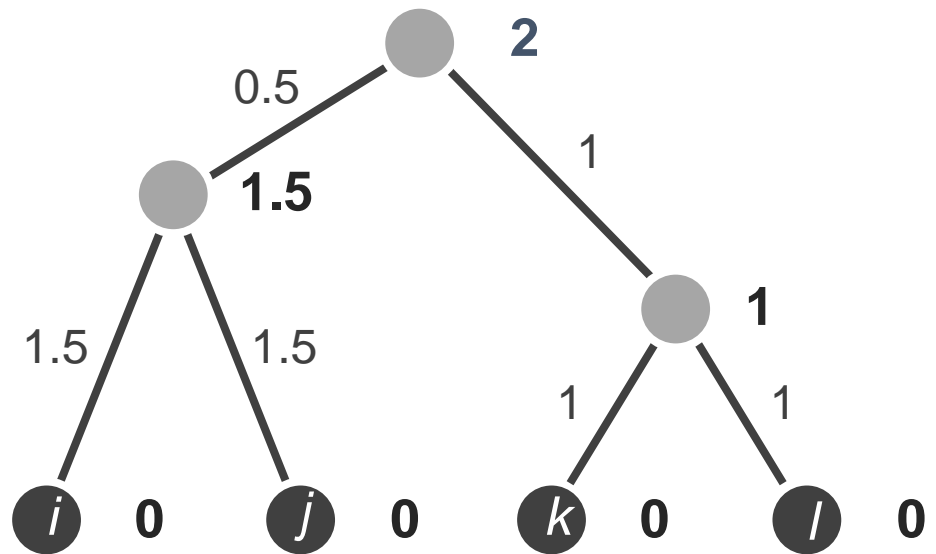
	$\{i, j\}$	$\{k, l\}$
$\{i, j\}$	0	4
$\{k, l\}$	4	0



UPGMA: heurističko klasterovanje

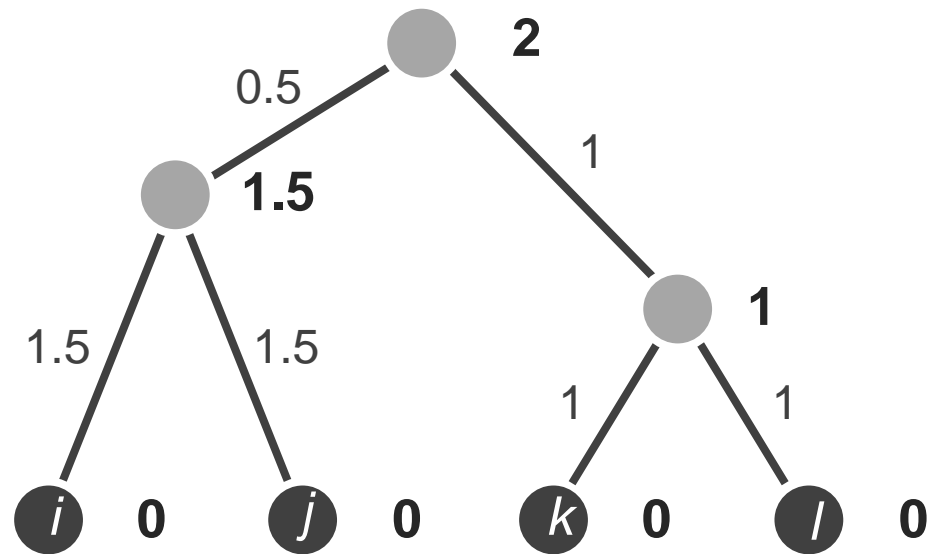
6. Iteriramo dok ne dođemo do jednog klastera koji sadrži sve vrste.

	$\{i, j\}$	$\{k, l\}$
$\{i, j\}$	0	4
$\{k, l\}$	4	0

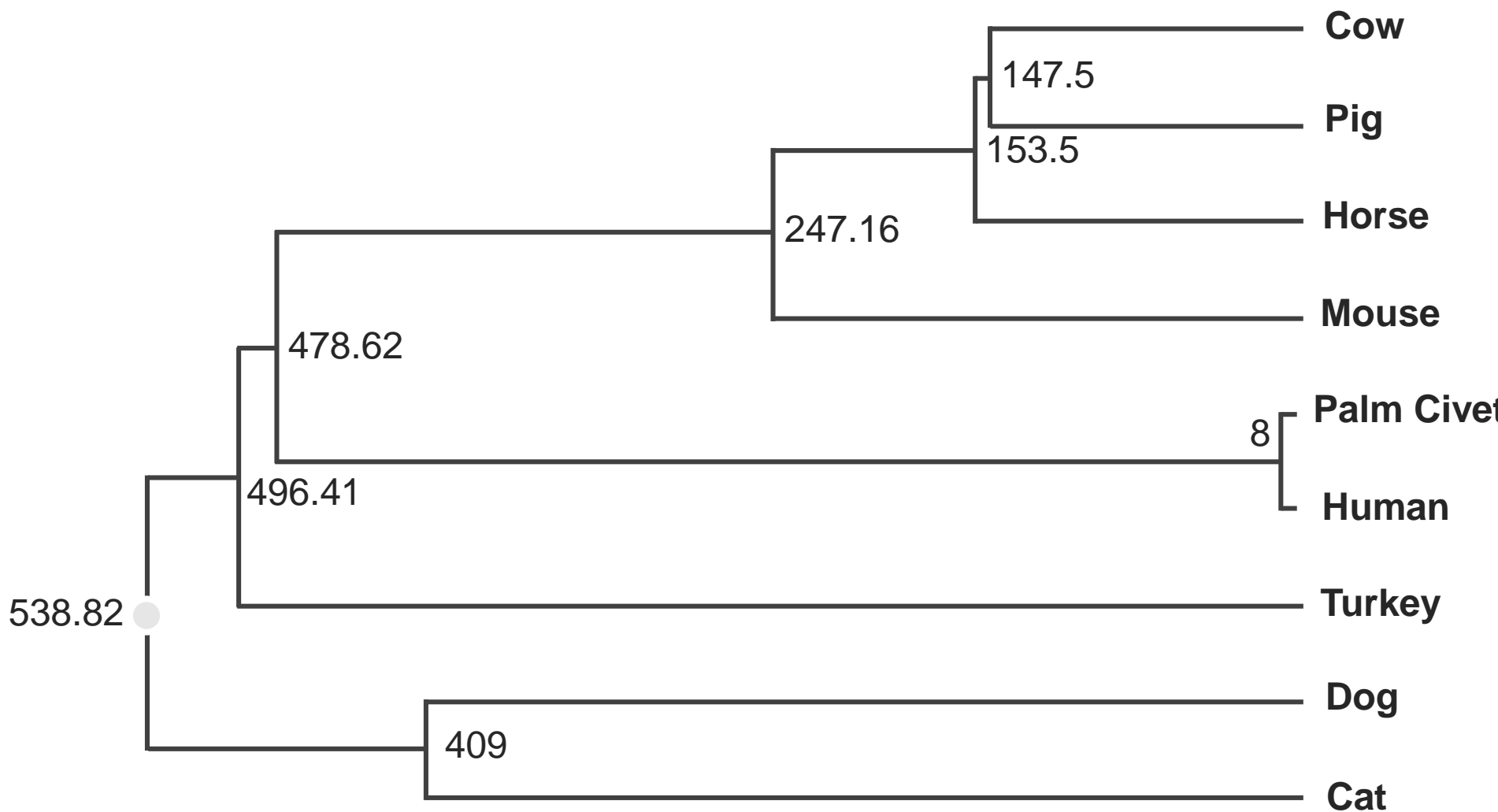


UPGMA: heurističko klasterovanje

6. Iteriramo dok ne dođemo do jednog klastera koji sadrži sve vrste.

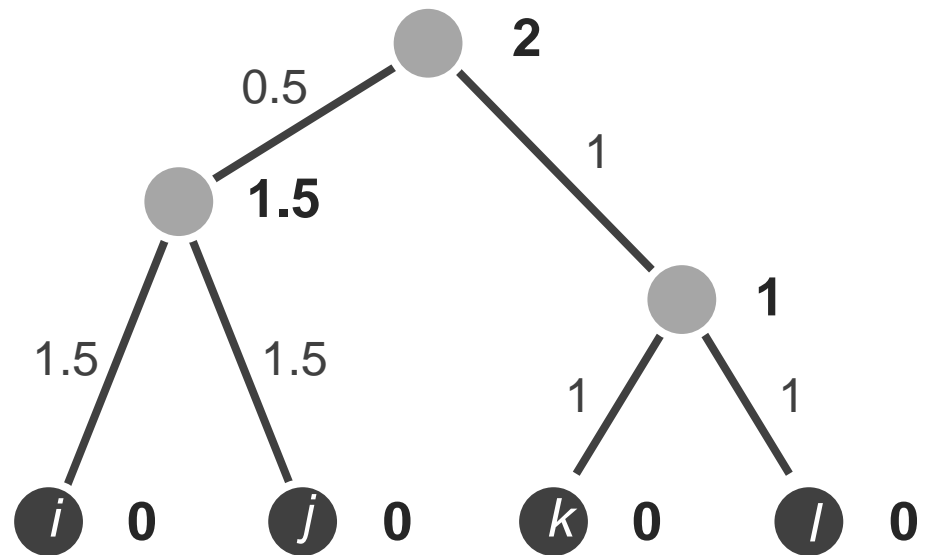


Primena UPGMA na *spike* proteine



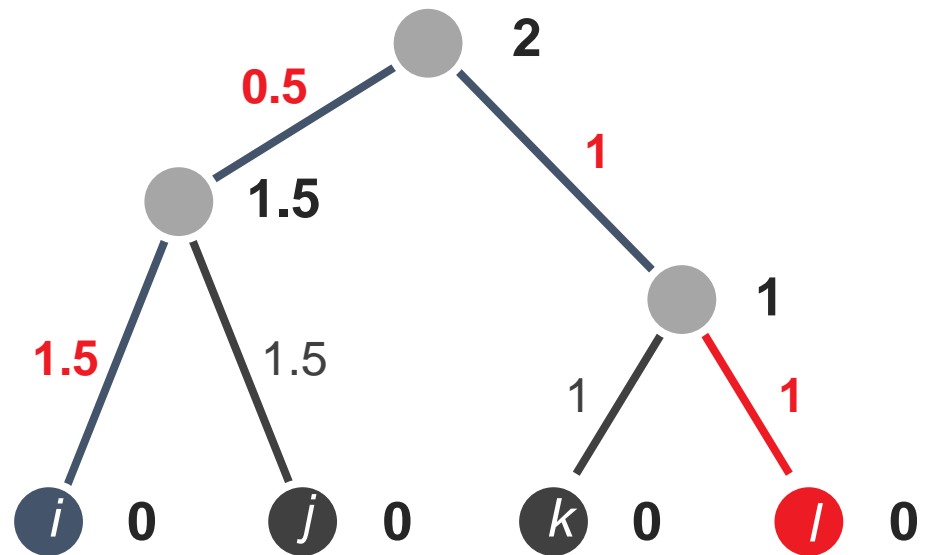
UPGMA ne proizvodi stablo koje odgovara matrici rastojanja

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0



UPGMA ne proizvodi stablo koje odgovara matrici rastojanja

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	0	3	4	3
<i>j</i>	3	0	4	5
<i>k</i>	4	4	0	2
<i>l</i>	3	5	2	0



Da rezimiramo...

- ***AdditivePhyLogeny***:
 - dobra strana: kreira stablo koje odgovara aditivnoj matrici
 - loša strana: ne radi za neaditivne matrice
- **UPGMA**:
 - dobra strana: kreira stablo za svaku matricu
 - loša strana: stablo ne mora da odgovara aditivnoj matrici
- **?????**:
 - dobra strana: kreira stablo koje odgovara aditivnoj matrici
 - dobra strana: heuristika za neaditivne matrice

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Neighbour-Joining teorema

Za datu matricu rastojanja D dimenzije $n \times n$, njena *neighbour-joining* matrica u oznaci D^* definiše se kao

$$D^*_{i,j} = (n-2) \cdot D_{i,j} - TotalDistance_D(i) - TotalDistance_D(j)$$

gde je $TotalDistance_D(i)$ suma rastojanja od i do svih ostalih listova.

		i	j	k	l	$TotalDistance_D$			i	j	k	l
D	i	0	13	21	22	56	D^*	i	0	-68	-60	-60
	j	13	0	12	13	38		j	-68	0	-60	-60
	k	21	12	0	13	46		k	-60	-60	0	-68
	l	22	13	13	0	48		l	-60	-60	-68	0

Neighbour-Joining teorema

Neighbour-joining teorema: ako je matrica D aditivna, onda minimalni element matrice D^* odgovara susednim listovima u stablu $Tree(D)$.

	i	j	k	l	$TotalDistance_D$		i	j	k	l	
D	i	0	13	21	22	56	i	0	-68	-60	-60
	j	13	0	12	13	38	j	-68	0	-60	-60
	k	21	12	0	13	46	k	-60	-60	0	-68
	l	22	13	13	0	48	l	-60	-60	-68	0

Neighbour-Joining algoritam

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>TotalDistance_D</i>
<i>D</i> *	0	-68	-60	-60	56
<i>j</i>	-68	0	-60	-60	38
<i>k</i>	-60	-60	0	-68	46
<i>l</i>	-60	-60	-68	0	48

1. Konstruišemo *neighbour-joining* matricu D^* na osnovu matrice D .

Neighbour-Joining algoritam

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>TotalDistance_D</i>
<i>i</i>	0	-68	-60	-60	56
<i>j</i>	-68	0	-60	-60	38
<i>k</i>	-60	-60	0	-68	46
<i>l</i>	-60	-60	-68	0	48

2. Nađemo minimalni element $D^*_{i,j}$ matrice D^* .

Neighbour-Joining algoritam

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>TotalDistance_D</i>		
<i>D</i> *	<i>i</i>	0	-68	-60	-60	56	$\Delta_{i,j} = (56 - 38) / (4 - 2)$ $= 9$
	<i>j</i>	-68	0	-60	-60	38	
	<i>k</i>	-60	-60	0	-68	46	
	<i>l</i>	-60	-60	-68	0	48	

3. Izračunamo $\Delta_{i,j} = (TotalDistance_D(i) - TotalDistance_D(j)) / (n - 2)$.

Neighbour-Joining algoritam

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>TotalDistance_D</i>	
<i>i</i>	0	13	21	22	56	
<i>j</i>	13	0	12	13	38	$\Delta_{i,j} = (56 - 38) / (4 - 2)$
<i>k</i>	21	12	0	13	46	$= 9$
<i>l</i>	22	13	13	0	48	

$$LimbLength(i) = \frac{1}{2}(13 + 9) = 11$$

$$LimbLength(j) = \frac{1}{2}(13 - 9) = 2$$

4. Postavimo $LimbLength(i)$ na $\frac{1}{2}(D_{i,j} + \Delta_{i,j})$ i $LimbLength(j)$ na $\frac{1}{2}(D_{i,j} - \Delta_{i,j})$.

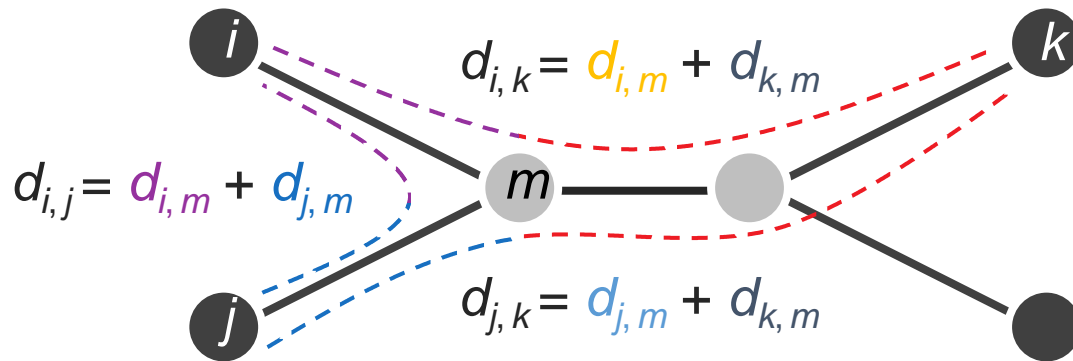
Neighbour-Joining algoritam

	<i>m</i>	<i>k</i>	<i>l</i>	<i>TotalDistance_D</i>
<i>m</i>	0	10	11	21
<i>D'</i> <i>k</i>	10	0	13	23
<i>l</i>	11	13	0	24

5. Formiramo matricu D' tako što uklonimo i -ti i j -ti red/kolonu iz D i dodamo m -ti red/kolonu tako da za svako k važi

$$D_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2.$$

Podsećanje: računanje of $d_{k,m}$

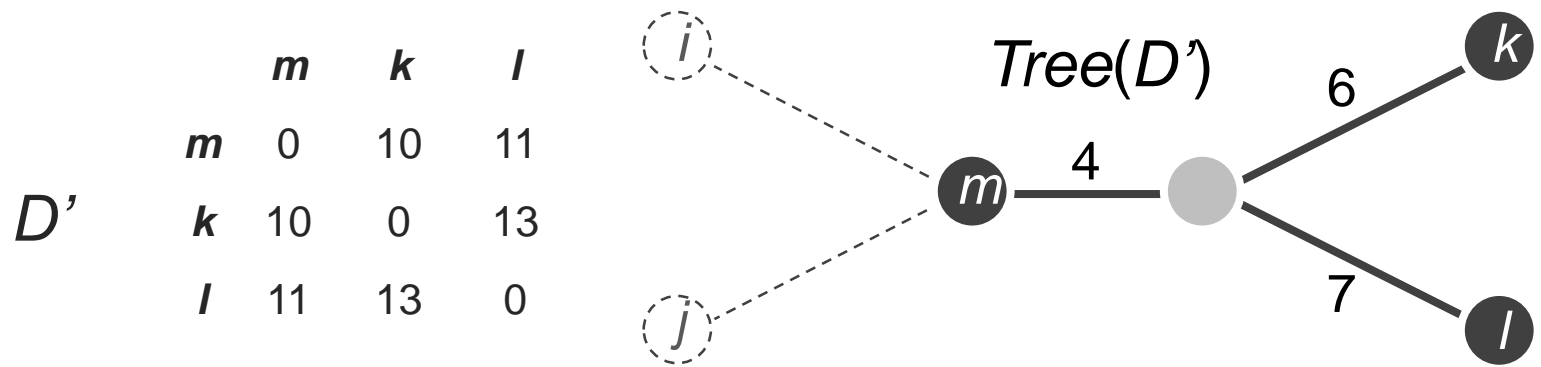


$$d_{k,m} = [(d_{i,m} + d_{k,m}) + (d_{j,m} + d_{k,m}) - (d_{i,m} + d_{j,m})] / 2$$

$$d_{k,m} = (d_{i,k} + d_{j,k} - d_{i,j}) / 2$$

$$d_{k,m} = (D_{i,k} + D_{j,k} - D_{i,j}) / 2$$

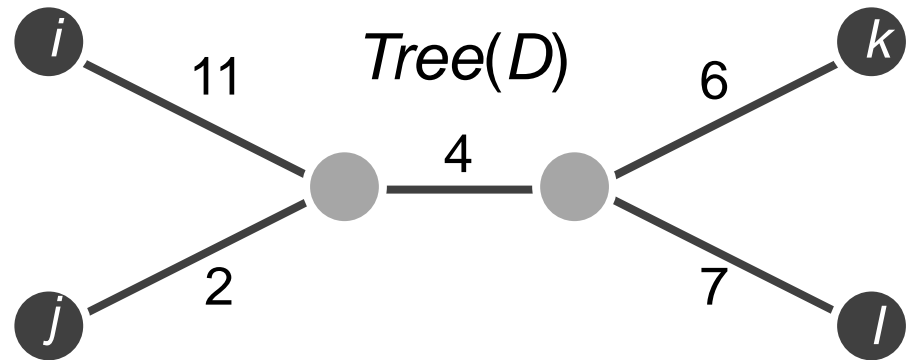
Neighbour-Joining algoritam



6. Primenimo *NeighborJoining* rekurzivno na *D'* da dobijemo *Tree(D')*.

Neighbour-Joining algoritam

	<i>m</i>	<i>k</i>	<i>l</i>
<i>m</i>	0	10	11
<i>k</i>	10	0	13
<i>l</i>	11	13	0



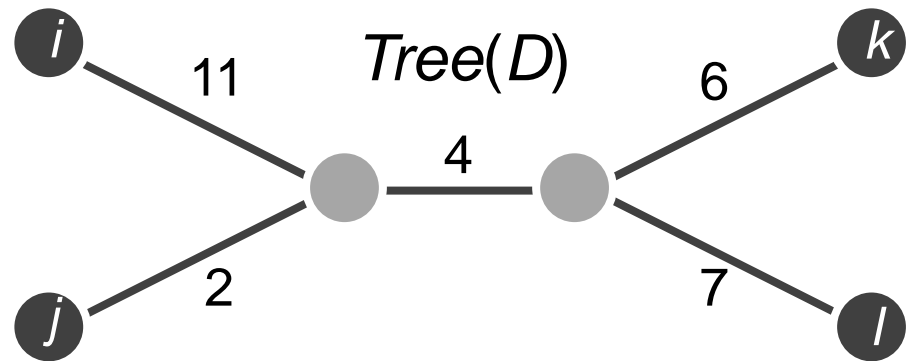
$$\text{LimbLength}(i) = \frac{1}{2}(13 + 9) = 11$$

$$\text{LimbLength}(j) = \frac{1}{2}(13 - 9) = 2$$

7. Vratimo krajnje grane do čvorova *i* i *j* i dobijemo *Tree(D)*.

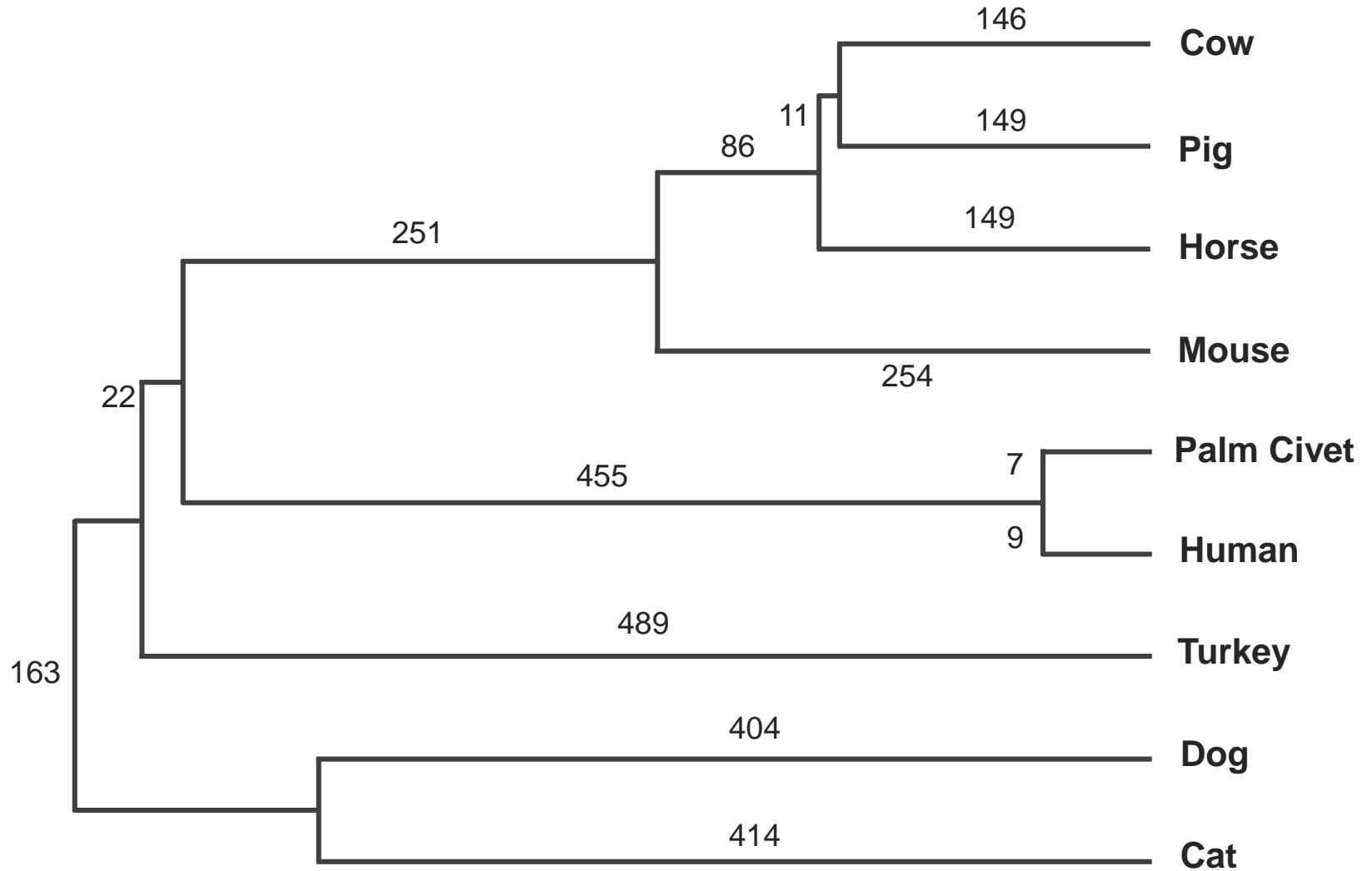
Neighbour-Joining algoritam

	<i>m</i>	<i>k</i>	<i>l</i>
<i>m</i>	0	10	11
<i>k</i>	10	0	13
<i>l</i>	11	13	0



7. Vratimo krajnje grane do čvorova *i* i *j* i dobijemo *Tree(D)*.

Neighbour-Joining za coronavirus



Slabosti metoda zasnovanih na rastojanju

Kada višestruko poravnanje zamenimo matricom rastojanja, gubimo informacije o sekvencama iz poravnanja

Zbog toga ne možemo da zaključimo kakva je sekvenca odgovarala vrstama iz unutrašnjih čvorova

SPECIES	ALIGNMENT	DISTANCE MATRIX			
		Chimp	Human	Seal	Whale
Chimp	ACGTAGGCCT	0	3	6	4
Human	ATGTAAGACT	3	0	7	5
Seal	TCGAGAGCAC	6	7	0	2
Whale	TCGAAAGCAT	4	5	2	0

Pregled

- Izbijanje epidemije
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- Problem male parsimonije
- Problem velike parsimonije

Tabele karakteristika

Pre oko pedeset godina, istraživači su konstruisali filogenetska stabla na osnovu anatomske-fizioloških osobina organizama koje su nazvane **karakteristikama**.



winged stick insect

wings **legs**

Yes 6



wingless stick insect

No 6



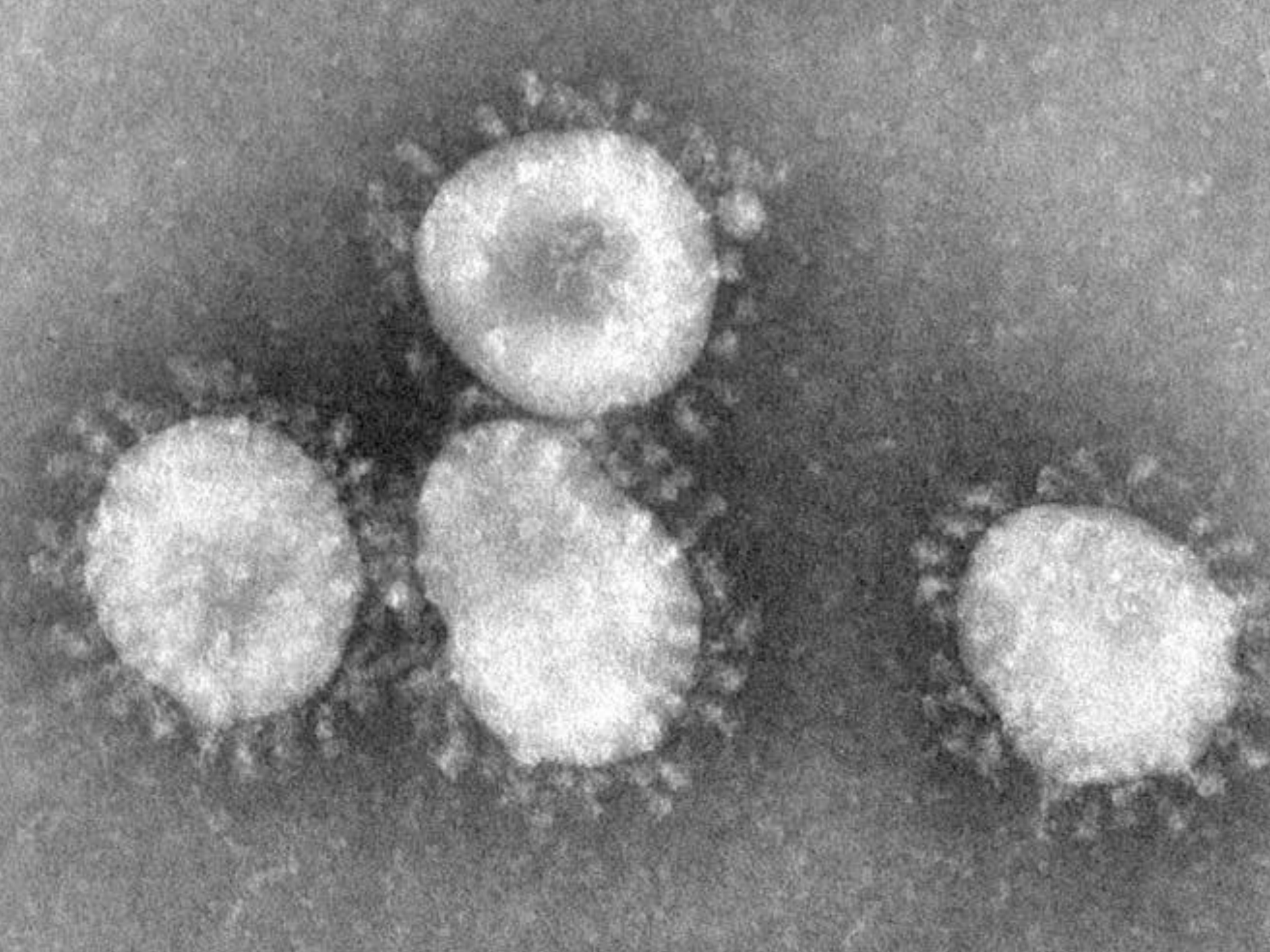
giant centipede

No 42

FiLOGENEZA NA OSNOVU KARAKTERISTIKA

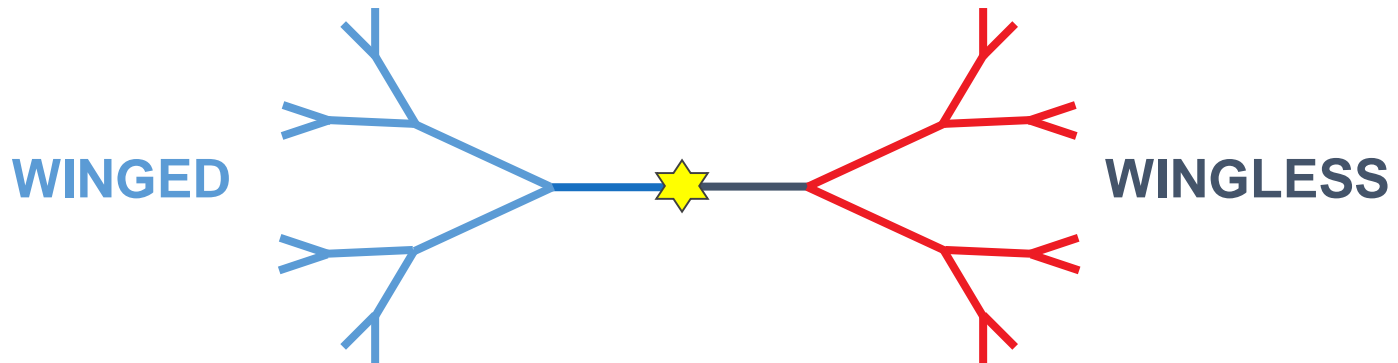
Problem filogeneze zasnovane na karakteristikama: *Rekonstruisati evolutivno stablo na osnovu karakteristika.*

- **Ulaz:** Tabela karakteristika $n \times m$ za n vrsta i m karakteristika.
- **Izlaz:** Stablo kod kog su vrste sa sličnim karakteristikama blizu jedna drugoj.



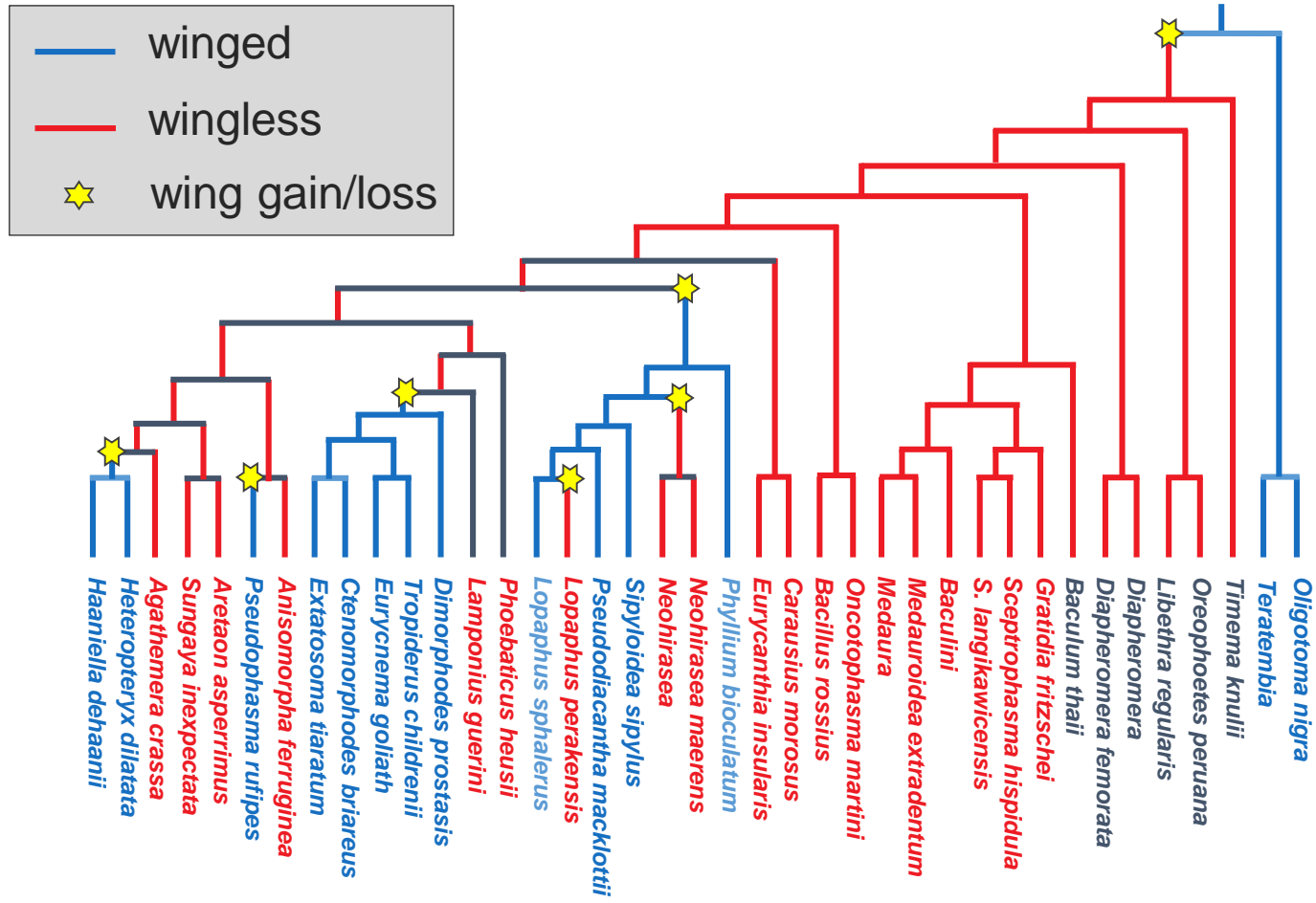
Od karakteristika do filogeneze

Kako bismo konstruisali evolutivno stablo na osnovu karakteristika?



Dolov zakon o nepovratnosti evolucionih procesa (1893): evolucija ne izmišlja dva puta isti organ (npr. krila kod insekata).

Izuzeci Dolovog zakona




Zašto je došlo do ovoga?

Poravnanje kao tabela karakteristika

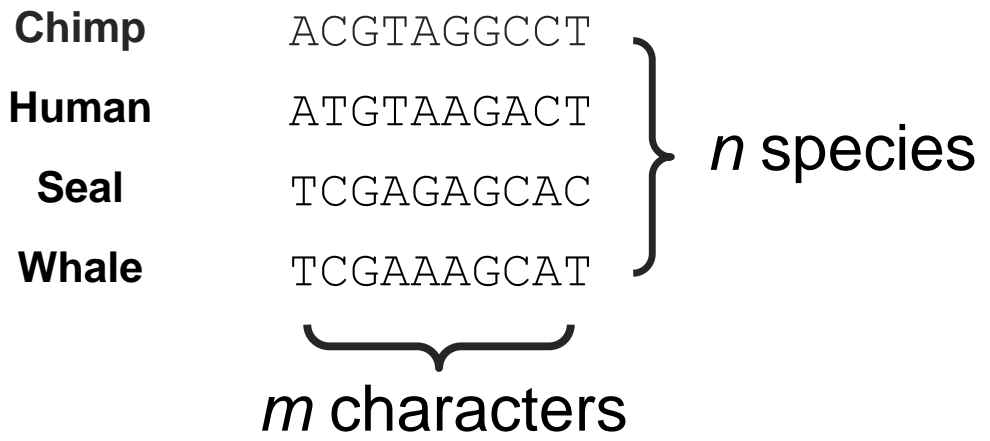
SPECIES	ALIGNMENT
Chimp	ACGTAGGCCT
Human	ATGTAAGACT
Seal	TCGAGAGCAC
Whale	TCGAAAGCAT

Poravnanje kao tabela karakteristika

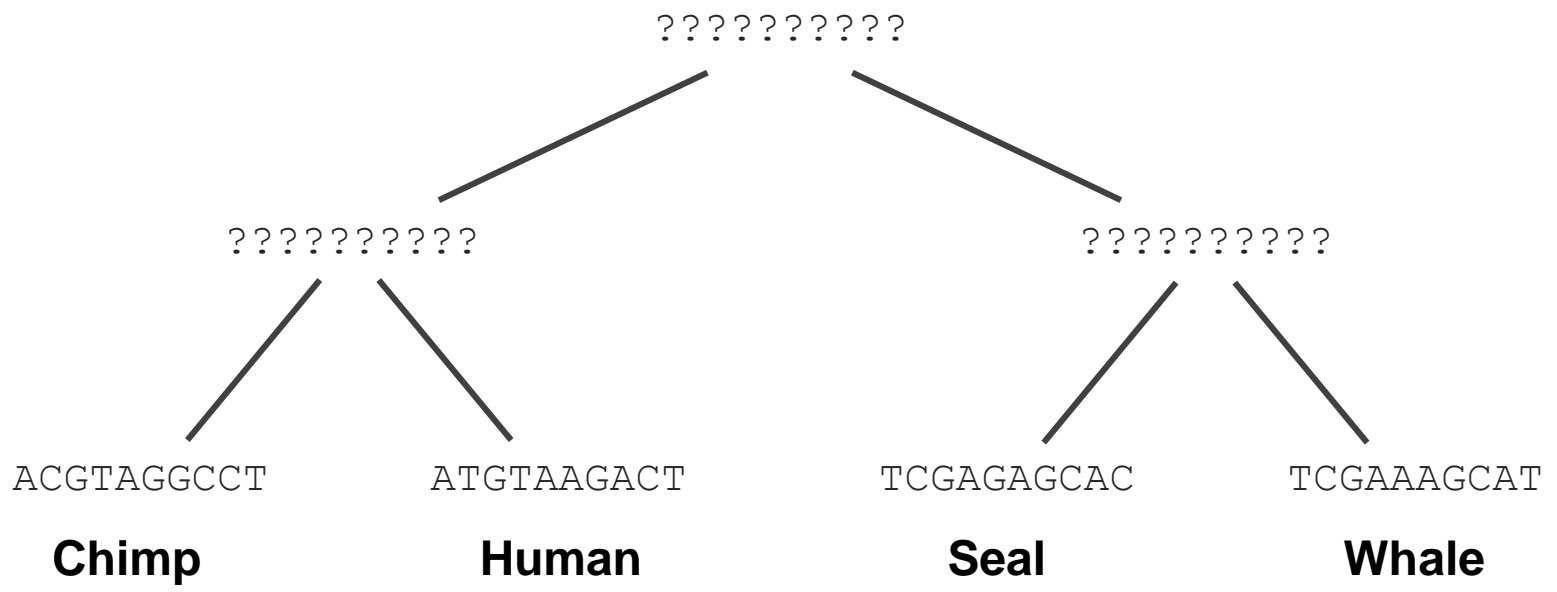
SPECIES	ALIGNMENT	
Chimp	ACGTAGGCCT	} <i>n</i> species
Human	ATGTAAGACT	
Seal	TCGAGAGCAC	
Whale	TCGAAAGCAT	
		
	<i>m</i> characters	

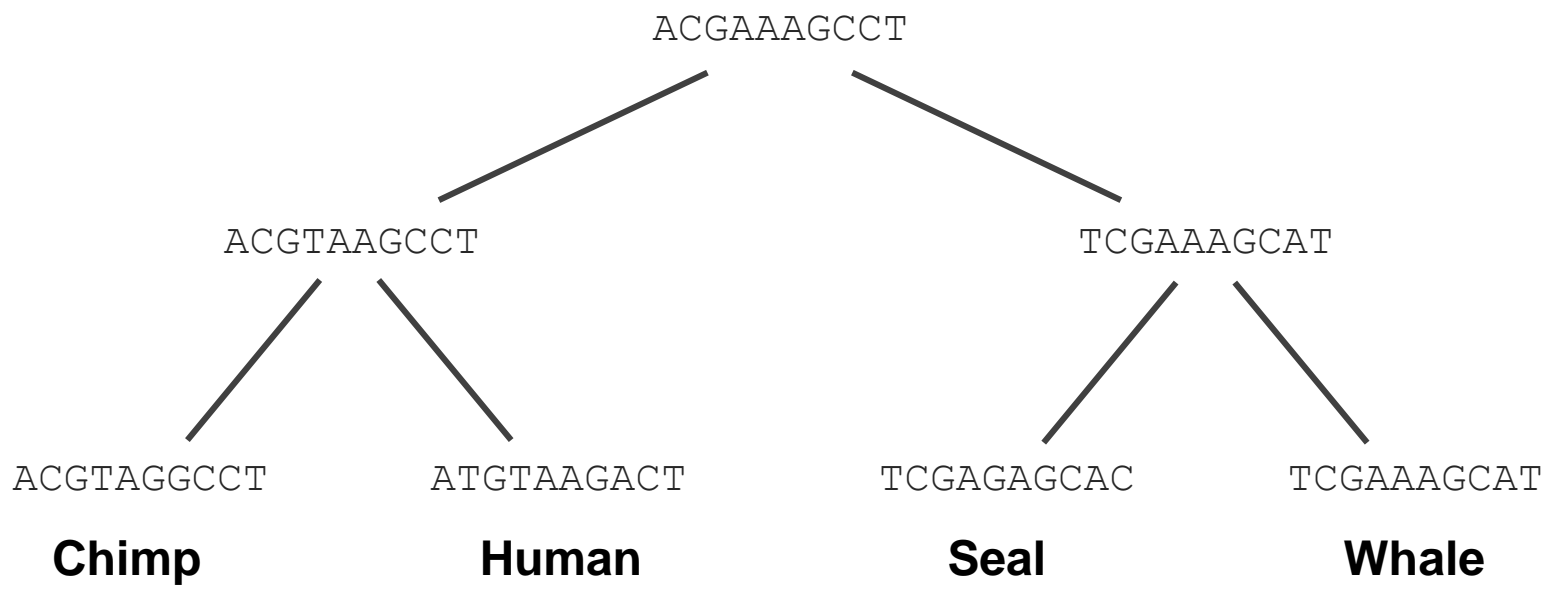
Pregled

- Izbijanje epidemije
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- Problem velike parsimonije



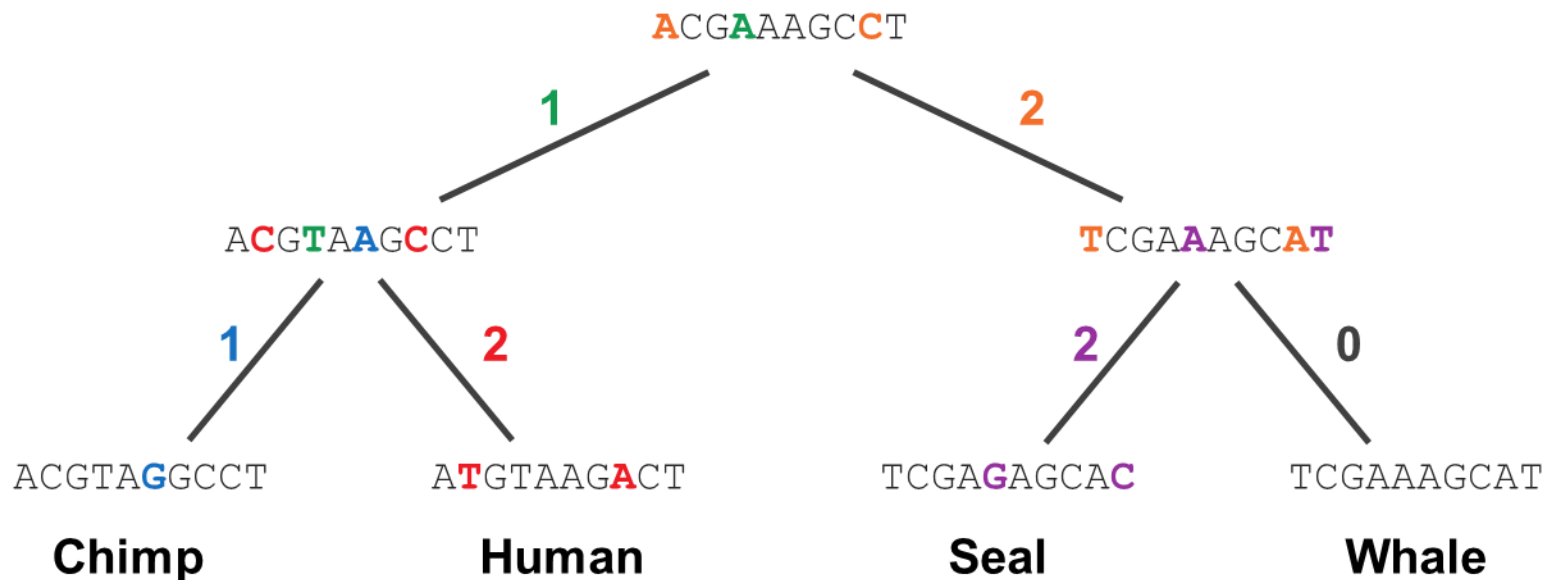
Chimp ACGTAGGCCT
Human ATGTAAGACT
Seal TCGAGAGCAC
Whale TCGAAAGCAT





Skor parsimonije: suma Hamingovih rastojanja duž svake grane.

Skor parsimonije: 8



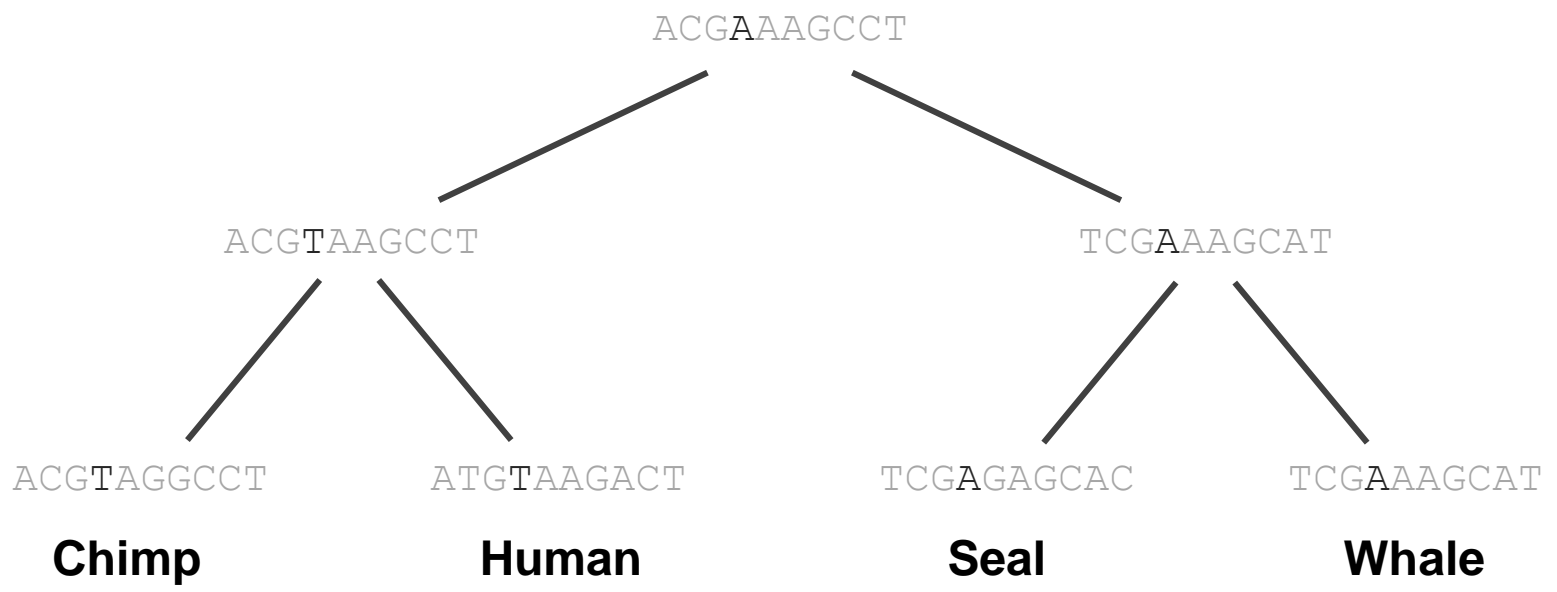
Problem male parsimonije: *Odrediti oznake za unutrašnje čvorove korenog stabla.*

- **Ulaz:** Koreno binarno stablo gde je svaki list označen niskom karaktera dužine m .
- **Izlaz:** Oznake za sve ostale čvorove stabla takve da minimizuju skor parsimonije stabla.

Da li ovu formulaciju možemo pojednostaviti?

Problem male parsimonije: *Odrediti oznake za unutrašnje čvorove korenog stabla.*

- **Ulaz:** Koreno binarno stablo gde je svaki list označen **jednim simbolom**.
- **Izlaz:** Oznake za sve ostale čvorove stabla takve da minimizuju skor parsimonije stabla.

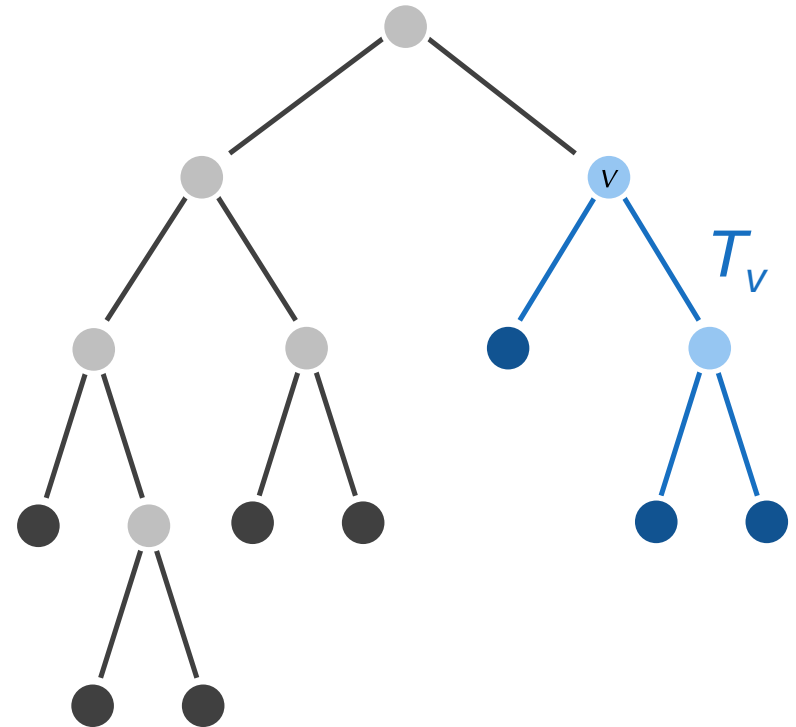


Algoritam dinamičkog programiranja

Neka je T_v podstablo stabla T sa korenom u čvoru v .

Neka je $s_k(v)$ minimalni skor parsimonije stabla T_v za sva moguća obeležavanja, pod pretpostavkom da je čvor v obeležen simbolom k .

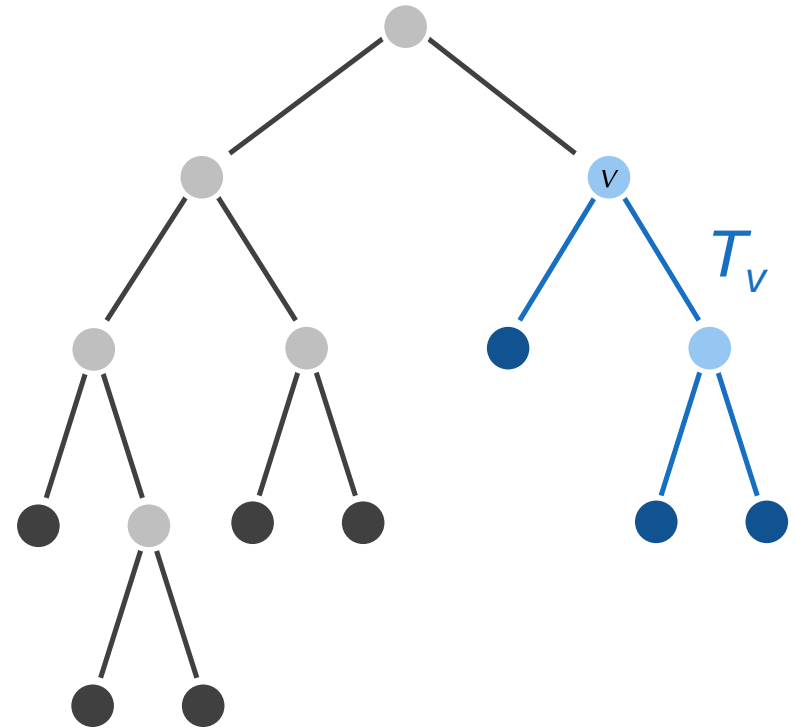
Minimalni skor parsimonije stabla jednak je minimalnoj vrednosti $s_k(\text{root})$ po svim simbolima k .



Algoritam dinamičkog programiranja

Neka je $\delta_{i,j}$ Kronekerov delta simbol:

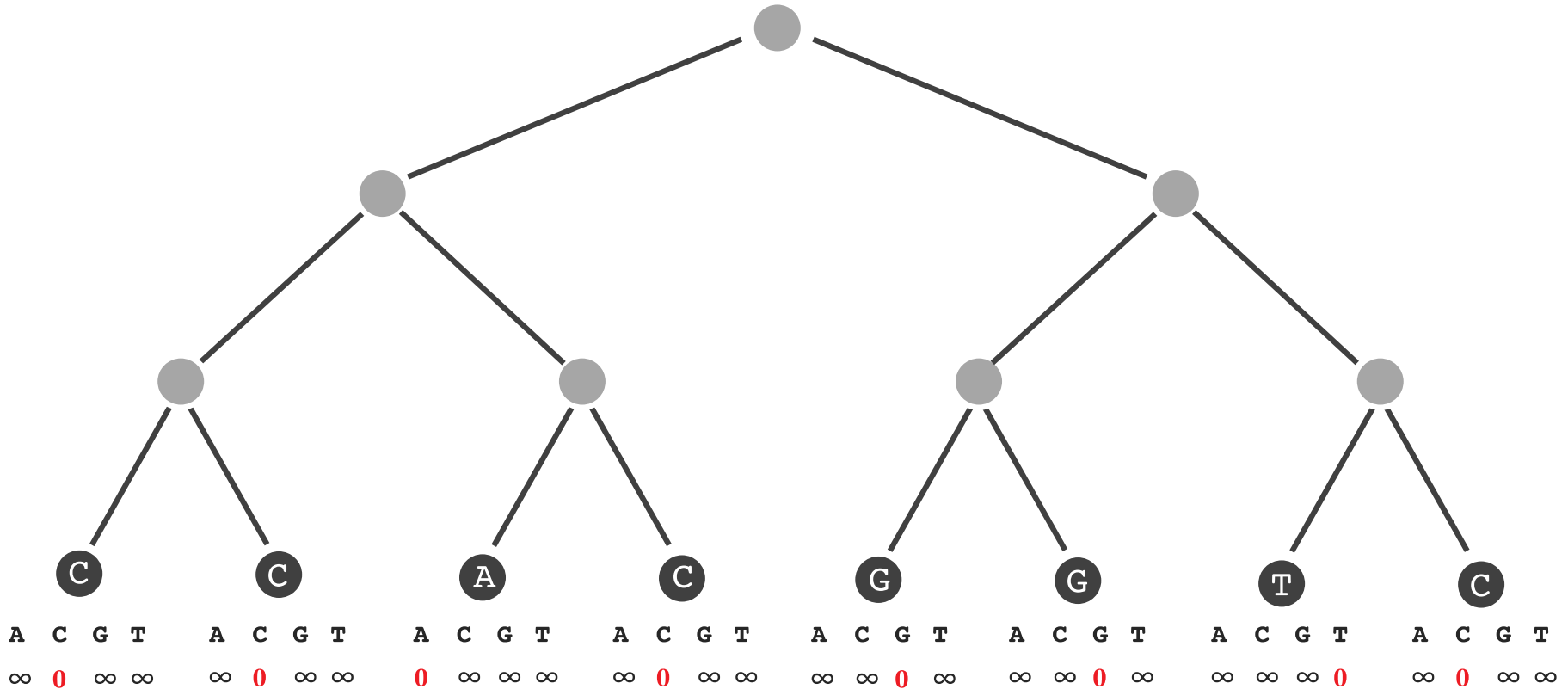
- $\delta_{i,j} = 0$ ako $i = j$
- $\delta_{i,j} = 1$ inače



Važi sledeća rekurentna relacija:

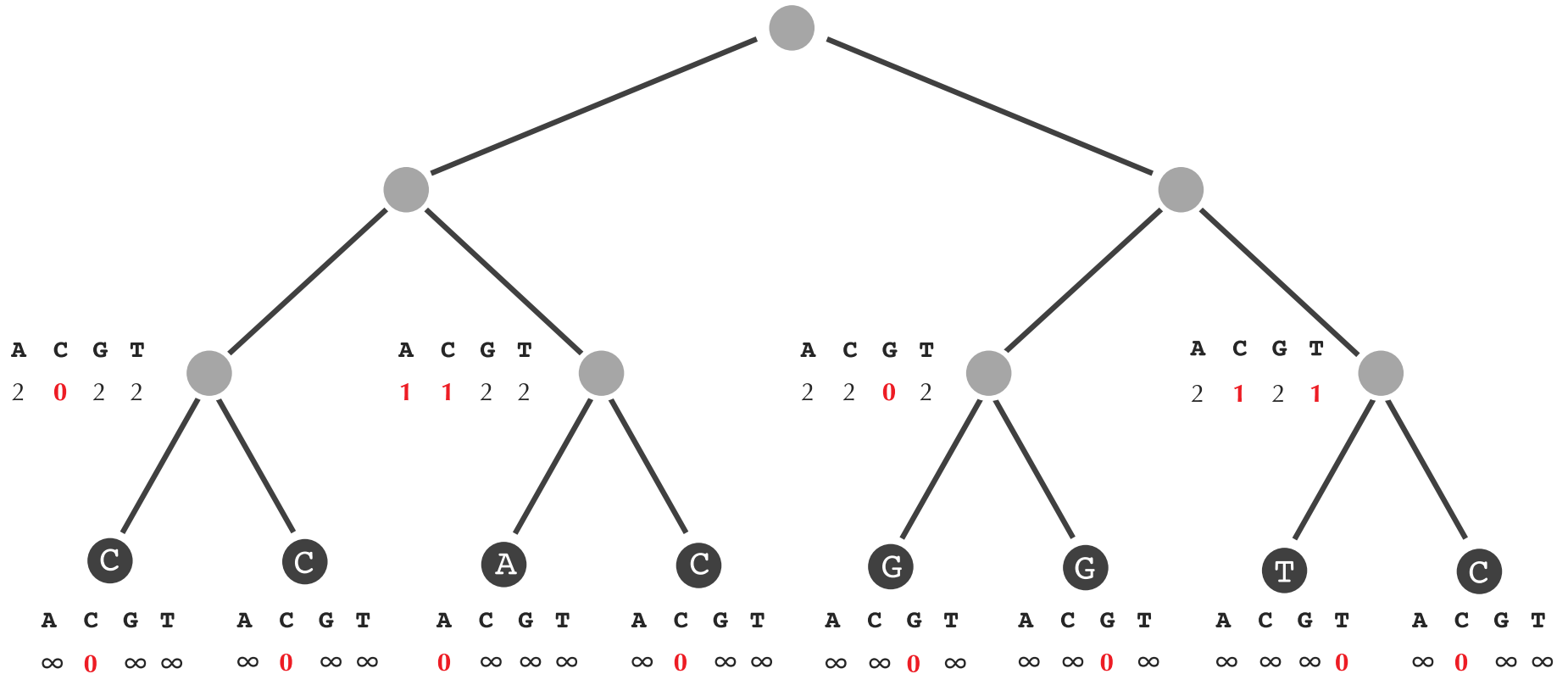
$$s_k(v) = \min_{\text{all symbols } i} \{s_i(\text{Daughter}(v)) + \delta_{i,k}\} + \min_{\text{all symbols } j} \{s_j(\text{Son}(v)) + \delta_{j,k}\}$$

Algoritam dinamičkog programiranja



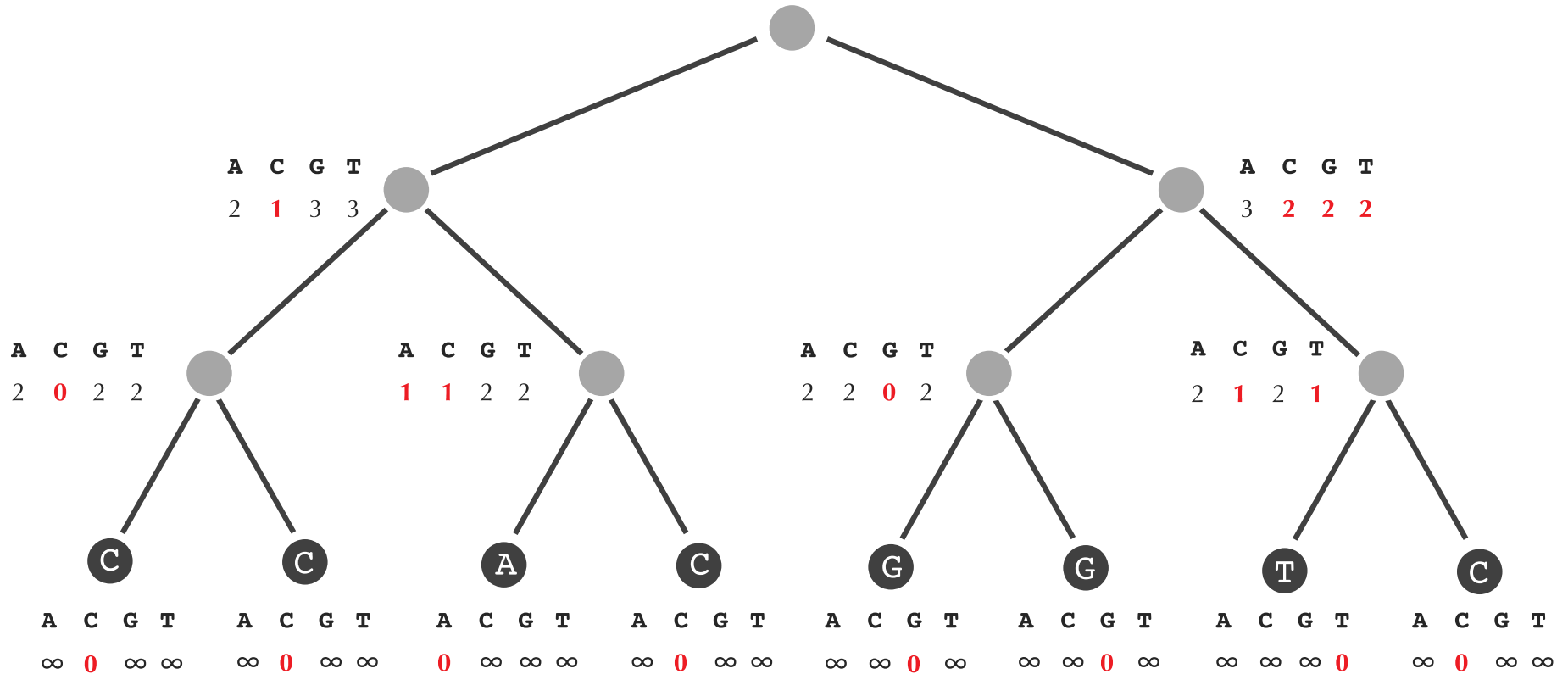
$$s_k(v) = \min_{\text{all symbols } i} \{s_i(\text{Daughter}(v)) + \delta_{i,k}\} + \min_{\text{all symbols } j} \{s_j(\text{Son}(v)) + \delta_{j,k}\}$$

Algoritam dinamičkog programiranja



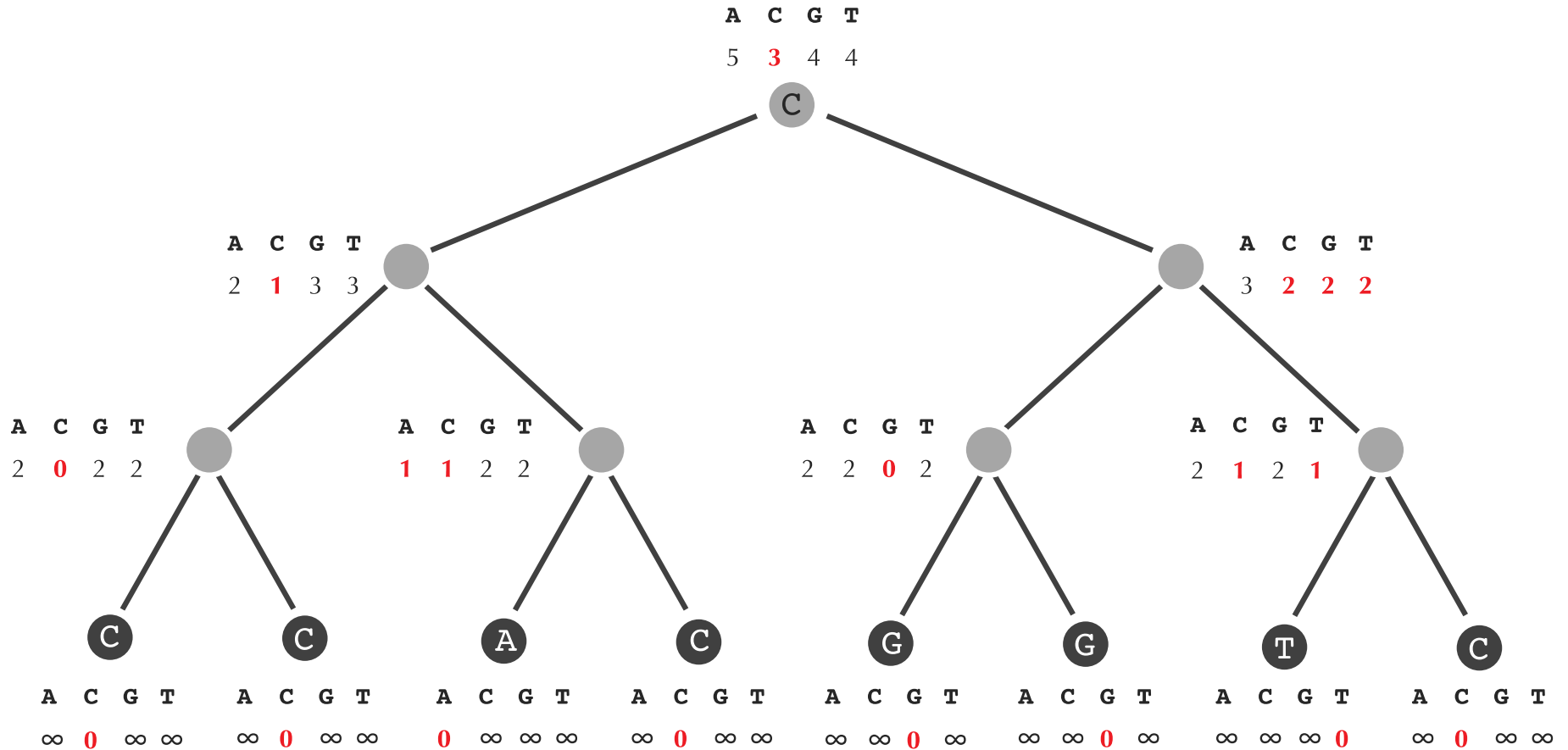
$$s_k(v) = \min_{\text{all symbols } i} \{s_i(\text{Daughter}(v)) + \delta_{i,k}\} + \min_{\text{all symbols } j} \{s_j(\text{Son}(v)) + \delta_{j,k}\}$$

Algoritam dinamičkog programiranja



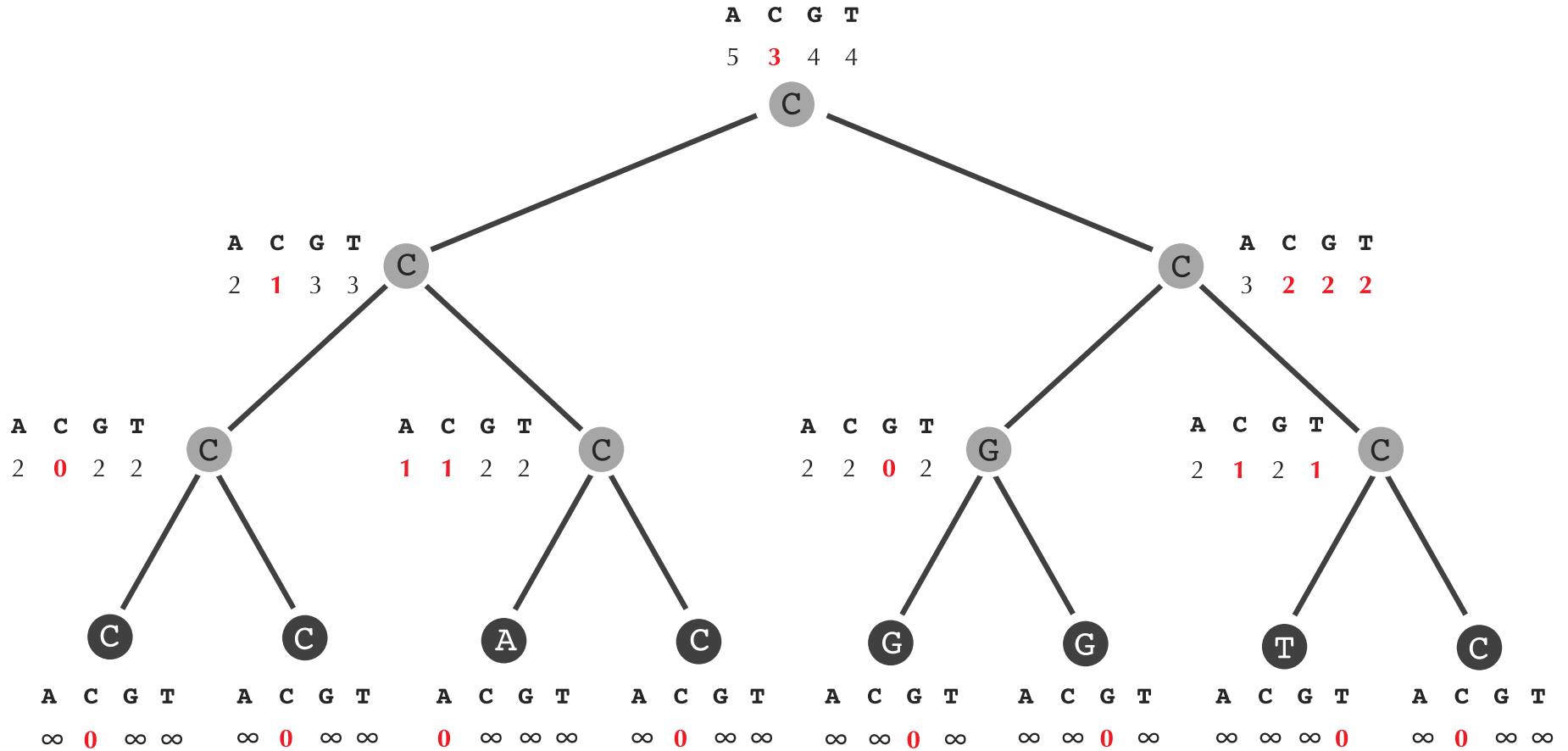
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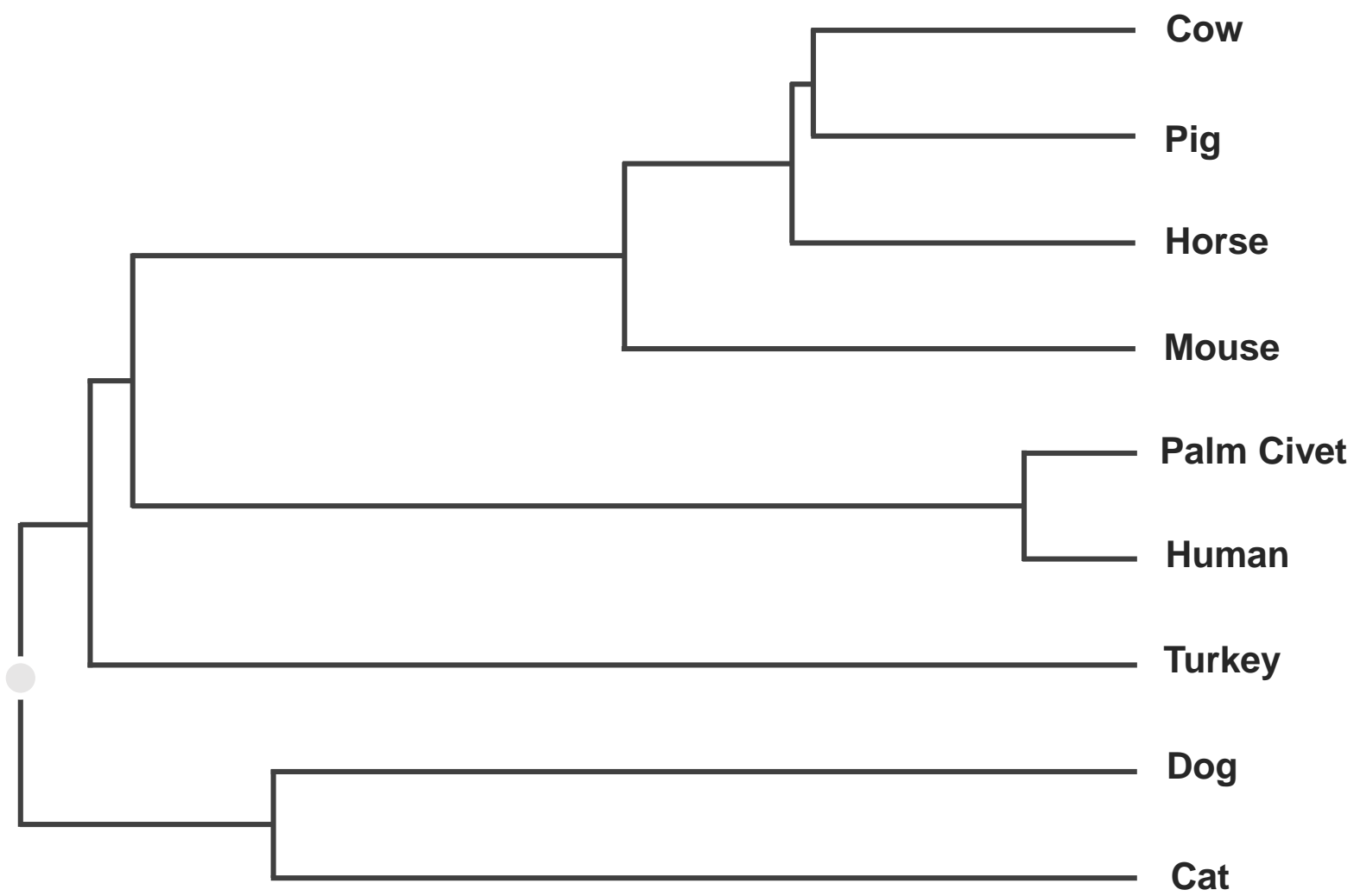
Algoritam dinamičkog programiranja



$$s_k(v) = \min_{\text{all symbols } i} \{s_i(\text{Daughter}(v)) + \delta_{i,k}\} + \min_{\text{all symbols } i} \{s_i(\text{Son}(v)) + \delta_{j,k}\}$$

Algoritam dinamičkog programiranja

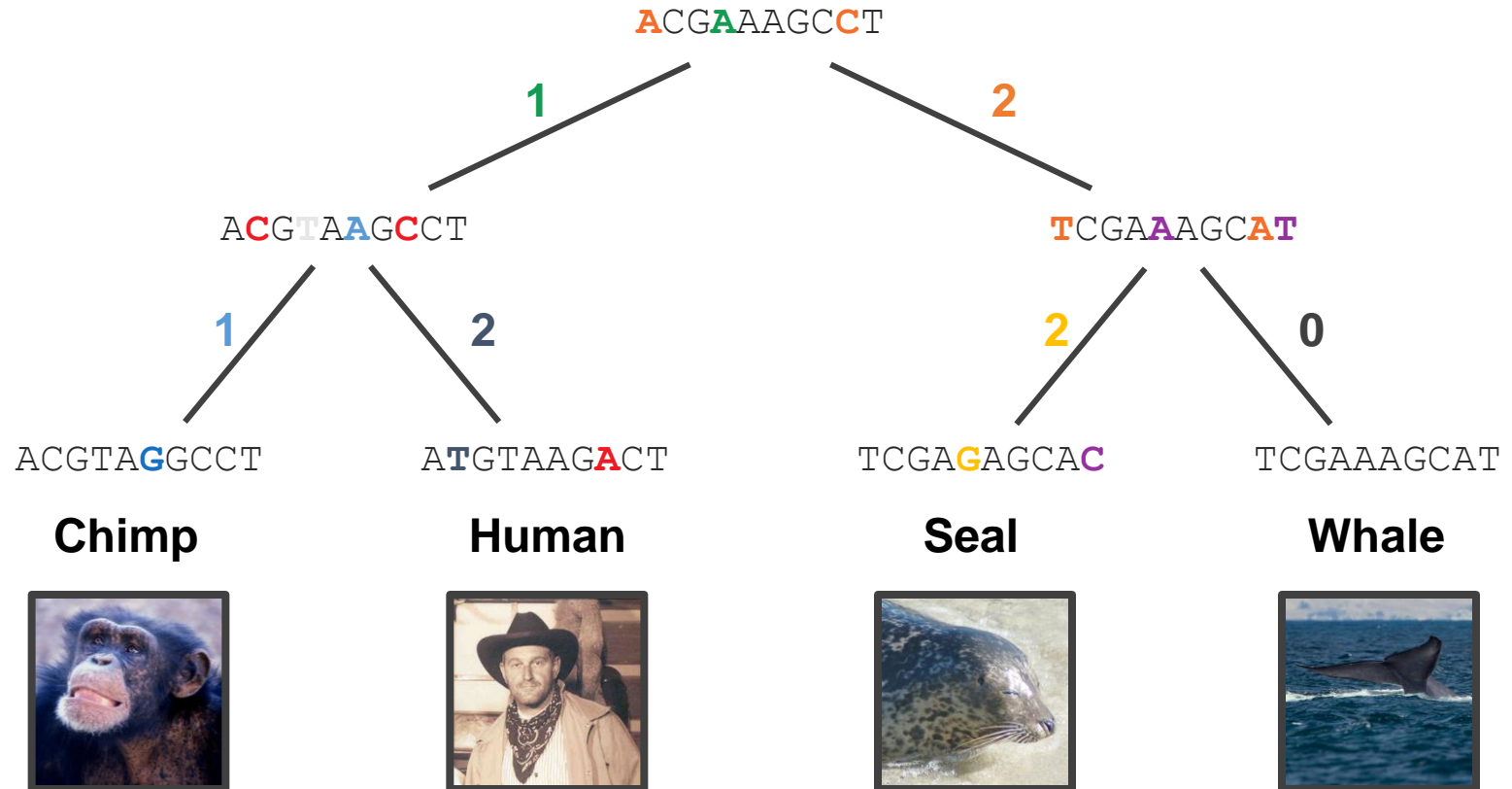




Pregled

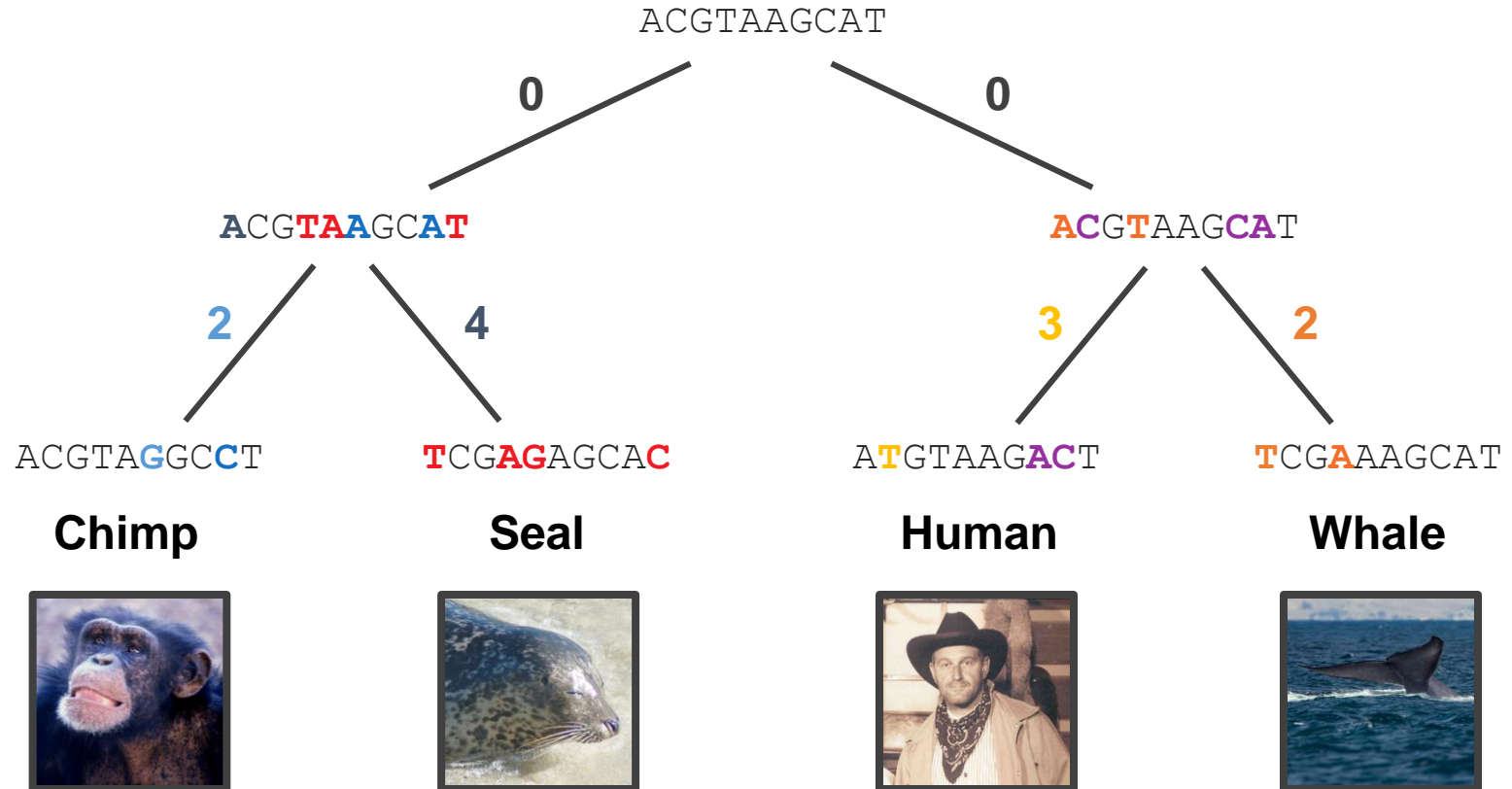
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Traženje odgovarajućeg stabla



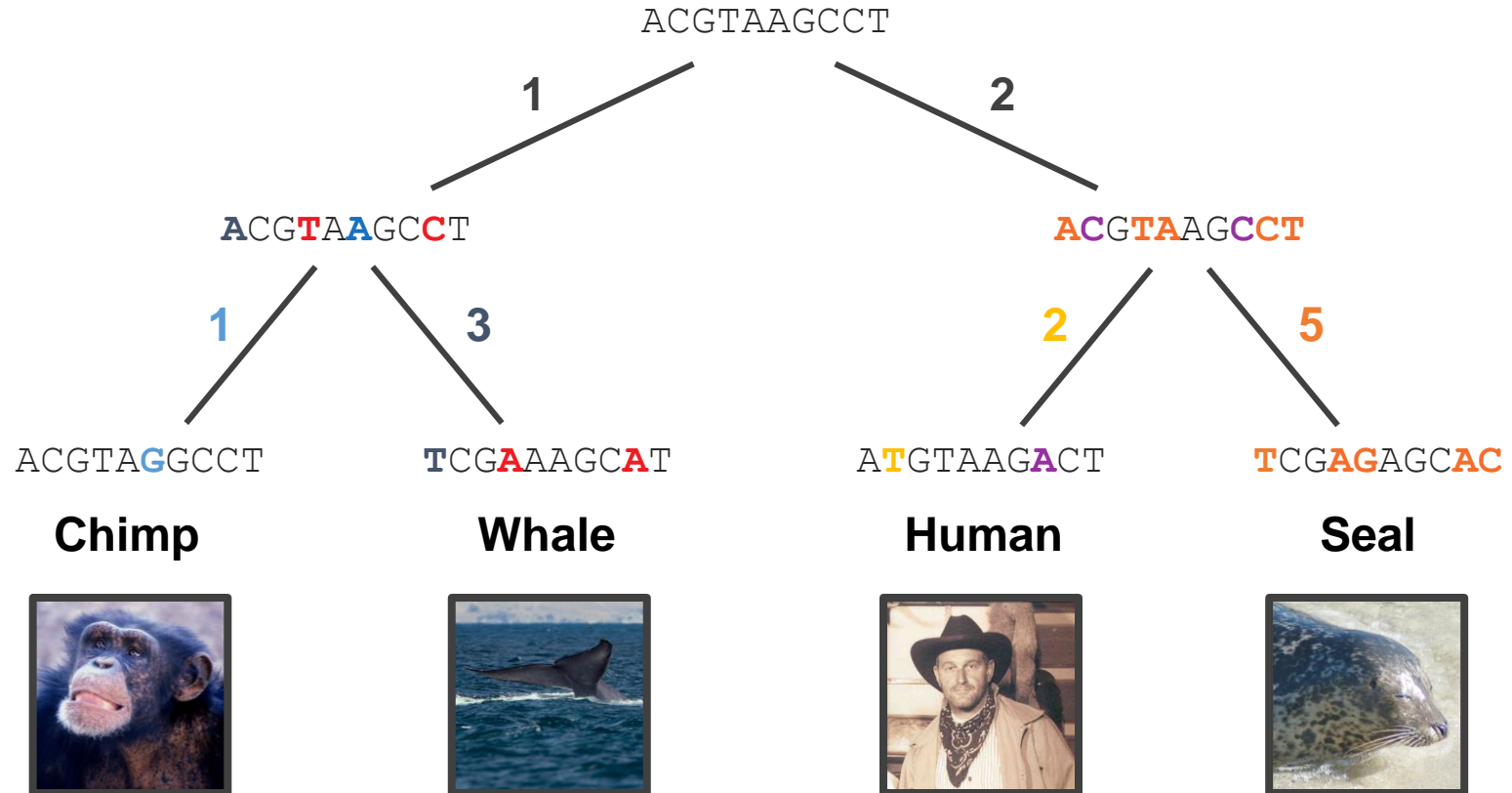
Skor parsimonije: 8

Traženje odgovarajućeg stabla



Skor parsimonije: 11

Traženje odgovarajućeg stabla



Skor parsimonije: 14

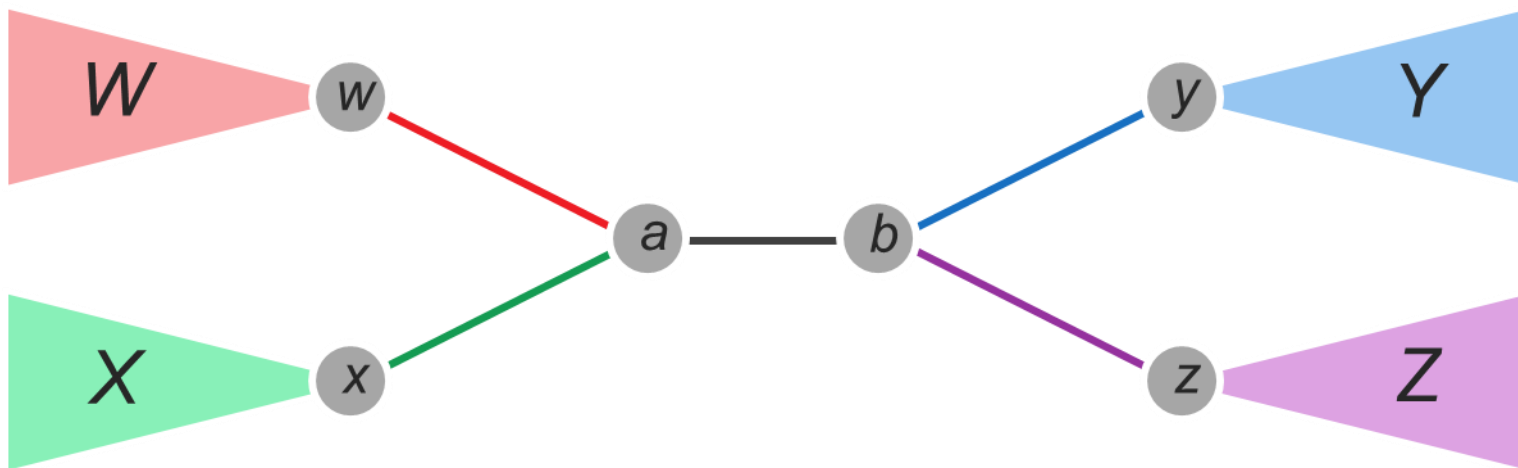
Problem velike parsimonije: *Za dati skup niski, naći stablo čiji su listovi označeni ovim niskama koje ima najmanji skor parsimonije.*

- **Ulaz:** Kolekcija niski jednake dužine.
- **Izlaz:** Koreno binarno stablo T koje minimizuje skor parsimonije po svim mogućim korenim binarnim stablima čiji su listovi označeni datim niskama.

Ovaj problem je NP-kompletan

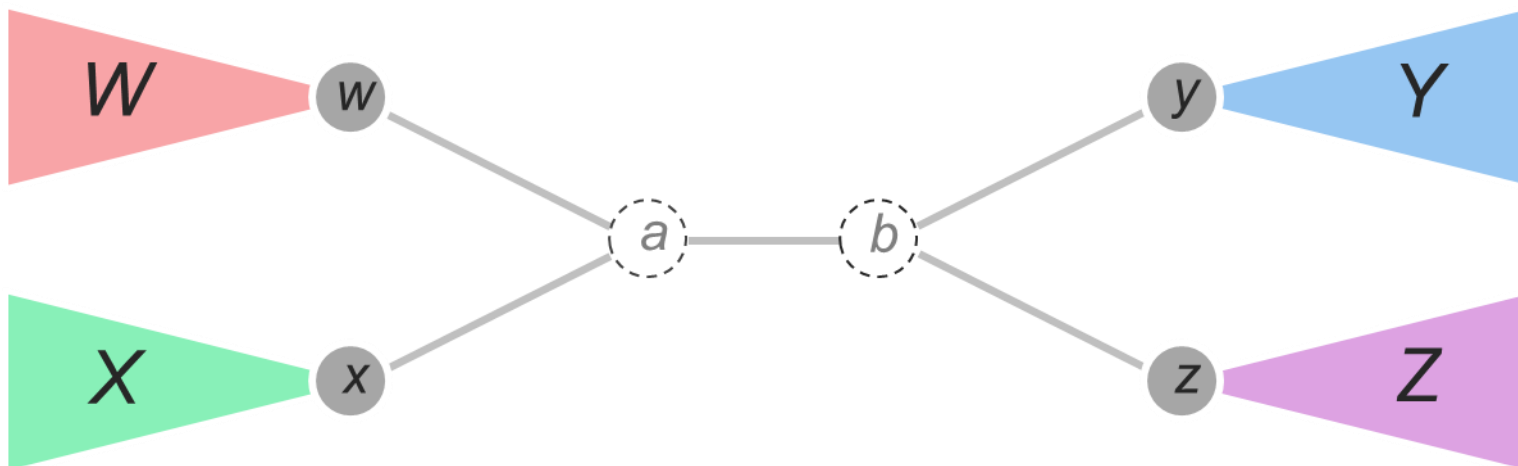
Pohlepna heuristika za veliku parsimoniju

Primetimo da uklanjanje jedne **unutrašnje grane**, grane koja povezuje dva unutrašnja čvora (zajedno sa čvorovima), dovodi do stvaranja četiri podstabla (W , X , Y , Z).



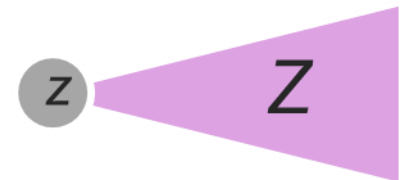
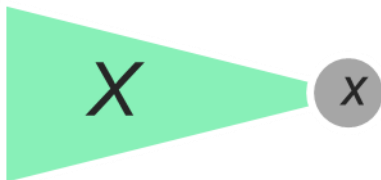
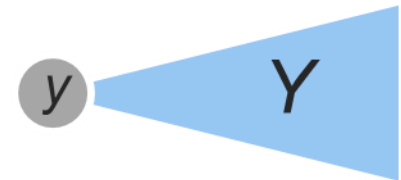
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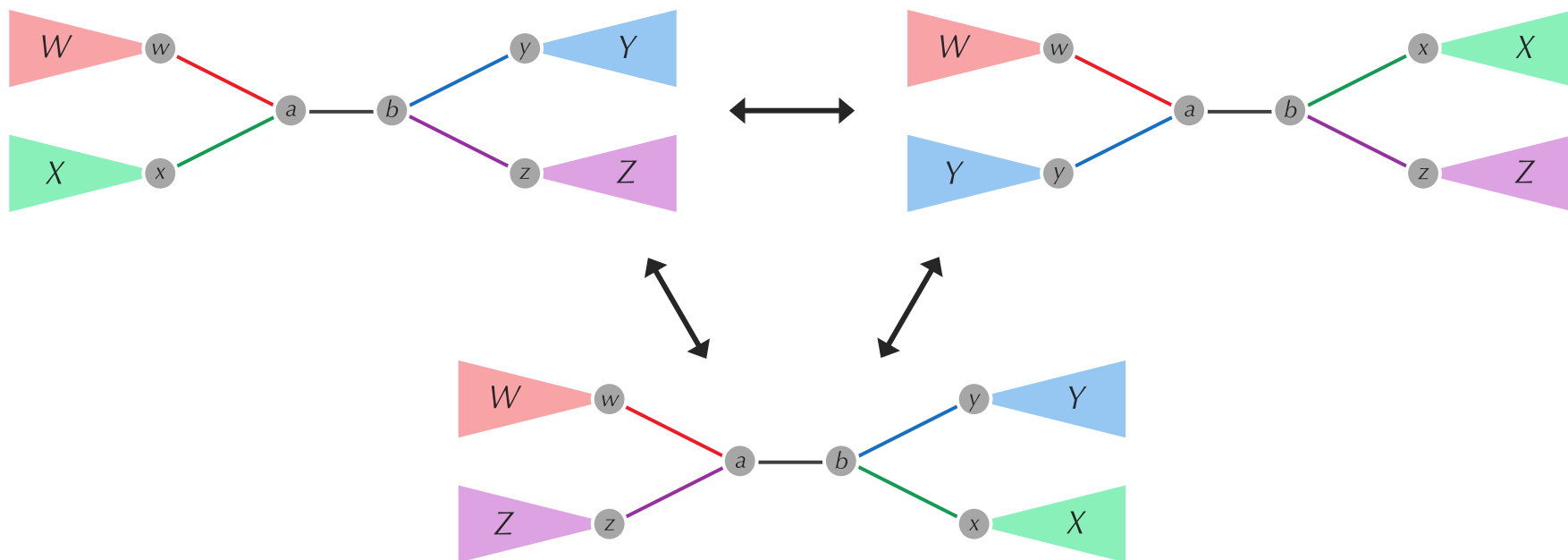
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Pohlepna heuristika za veliku parsimoniju

Preuređenje rasporeda ovih podstabala se naziva **razmena najbližih suseda**.



Pohlepna heuristika za veliku parsimoniju

Problem najbližih suseda u stablu: *Za datu granu u binarnom stablu, generisati dva suseda ovog stabla.*

- **Ulaz:** Unutrašnja grana binarnog stabla.
- **Izlaz:** Dva najbliža suseda ovog stabla za datu unutrašnju granu.

Pohlepna heuristika za veliku parsimoniju

Heuristika za razmenu najbližih suseda:

1. Postaviti trenutno stablo na koreno binarno stablo proizvoljne strukture

Pohlepna heuristika za veliku parsimoniju

Heuristika za razmenu najbližih suseda:

1. Postaviti trenutno stablo na koreno binarno stablo proizvoljne strukture
2. Proći kroz sve unutrašnje grane i izvršiti sve moguće razmene najbližih suseda

Pohlepna heuristika za veliku parsimoniju

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3. Rešiti problem male parsimonije za svako takvo stablo

Pohlepna heuristika za veliku parsimoniju

Heuristika za razmenu najbližih suseda:

1. Postaviti trenutno stablo na koreno binarno stablo proizvoljne strukture
2. Proći kroz sve unutrašnje grane i izvršiti sve moguće razmene najbližih suseda
3. Rešiti problem male parsimonije za svako takvo stablo
4. Ako stablo ima skor parsimonije bolje od optimalnog stabla, postaviti da to bude trenutno stablo; inače, vratiti trenutno stablo

- Slajdovi pokrivaju poglavlje 7 knjige *Bioinformatics Algorithms: an Active Learning Approach*
- Sadržaj slajdova je preuzet sa zvaničnih prezentacija autora i dodatno prilagođen